Introduction

Recently, a new iterative method, named Marchenko redatuming, was introduced to retrieve the Green’s function with a virtual receiver located in the subsurface, which begins with the original work on inverse scattering theory as formulated by Gel’fand, Levitan, and Marchenko (Gel’fand and Levitan, 1951; Marchenko, 1955). The Green’s functions can be retrieved from focusing functions and the dataset measured at the acquisition surface (Broggini and Snieder, 2012; Wapenaar et al., 2014a). By deconvolving the retrieved Green’s functions, we can obtain the virtual reflection response. This is the basis for obtaining the artifact-free image. With the knowledge of the direct arrival travel time between the surface and a subsurface point, which typically requires the use of a macro velocity model, the Marchenko imaging can be implemented (Wapenaar et al., 2014b; Singh et al., 2016; Thorbecke et al., 2017; Lomas et al., 2018; Jia et al., 2017; Brackenhoff et al., 2019). This imaging scheme is target-oriented; it can move the sources and receivers down to some specified locations inside the medium, which provides a good solution to study the area below highly complex media such as salt domes (Vasconcelos et al., 2019).

In standard migration images, artifacts appear because of internal multiple reflections. Several strategies have been introduced to predict and remove internal multiple reflections from measured data before the imaging processing, such as internal multiple elimination (Berkhout and Verschuur, 2005) and inverse scattering series (Weglein et al., 1997). Marchenko imaging has been developed to image without the disturbance of internal multiple reflections (Slob et al., 2014; Wapenaar et al., 2014b). This technique is mostly data-driven as it simply requires a smooth background velocity model, which is used to compute an estimate of the direct travel time from the virtual source (or to the virtual receiver). Thorbecke et al. (2013) have a sensitivity analysis of Green’s function retrieval and show that this estimate of the direct arrival does not have to be accurate and it is robust. To make this Marchenko-based scheme totally model-free, a data-driven internal multiples elimination scheme has been proposed by Van der Neut and Wapenaar (2016) and Zhang and Staring (2018) to process the original dataset. The method can provide a cleaner image during the imaging process (Zhang et al., 2019). This alternative Marchenko-based imaging scheme fits the conventional reverse time migration very well as it just removes the internal multiples in the original dataset and uses the retrieved data set for RTM imaging.

In this abstract, we will first review the basic scheme of the retrieved Green’s functions and the imaging condition of Marchenko imaging. Then we will discuss the dependence of the accuracy of estimated travel time field in the process of Marchenko imaging and some potential applications.

Method

Green’s function retrieval

Thorbecke et al. (2017) give us the definition of the iterative Marchenko scheme as follows,

\[ f_{i,k} = G_i^* + M_i^* \]

\[ f_{i,k} = f_{i,0} + \Theta R M_i^* \]

Where \( G_i^* \) is the time reversal of the direct arrival of Green’s function. \( M_i^* \) is the \( k^{th} \) iterative steps of scattering coda. \( \Theta \) is a time truncation to exclude values outside of the window. \( f_{i,0} \) and \( f_{i,k} \) is the decomposed upgoing (-) and downgoing (+) focusing function. With equation (1) and (2), we can get the retrieved Green’s function \( G^{**} \) (Wapenaar et al., 2014b). Here, we still need the information about direct arrival travel time. Usually, we use a macromodel to get the travel time field.

There is also an alternative way to implement data set retrieval. Based on Van der Neut and Wapenaar (2016), the focusing functions for focusing points at a specified depth are projected back to the acquisition surface. The idea is modified by Zhang and Staring (2018), which named the Marchenko multiple elimination scheme, then the retrieved data set can be given as

\[ R(x'_0, x''_0, t) = R(x'_0, x''_0, t) + \sum_{m=1}^{\infty} (\Theta \Theta R \Theta R)^m R(x'_0, x''_0, t) \]
Where \( x'_0 \) denotes the source position and \( x'_0 \) denotes the receiver position. \( R_0 \) is the retrieved data set with no internal multiples. \( R \) indicates the acoustic impulse reflection response, it’s a dataset measured at acquisition surface without surface related multiples. \( \mathcal{R} \) denotes a convolution integral operator of the measured data \( R \), \( \mathcal{R}^* \) is a correlation integral operator. We can consider the equation (3) as a process that eliminates the internal multiples from the original data set just in this formation \( R_0 = R - R_M \). This scheme has no model needed. The retrieved data set are multiple free and is more suitable for standard imaging and inversion than the original data. This provides a good way to remove the multiples during the process of seismic imaging and inversion.

**Imaging**

The estimated Green’s functions can be used to create images with the knowledge of direct arrival travel time. In theory, this scheme is able to calculate more accurate images than standard methods because the retrieved Green’s functions \( G^+ \) are more accurate, which accounts for the internal multiples. Here, we use the imaging condition of Marchenko imaging proposed by da Costa Filho et al. (2015), which is a zero-time lag cross-correlation between the direct arrival estimate \( T_d \) and the downgoing Green’s function \( G^+ \).

\[
I_{\text{Marchenko}}(x_i) = \sum_{x_0} \int_{-\infty}^{\infty} T_d(x_i, x_0, t) G^+(x_0, x_i, t)dt
\]

In order to compare the Marchenko imaging with the conventional one, we adopt another imaging condition to approximate standard methods (da Costa Filho and Curtis, 2016):

\[
I_{\text{RTM}}(x_i) = \sum_{x_0} \int_{-\infty}^{\infty} T_d(x_i, x_0, t) G_{\text{RTM}}(x_0, x_i, t)dt
\]

Where \( G_{\text{RTM}} \) is the back-propagated wavefield used in RTM, which is approximated as \( G_{\text{RTM}} \approx R \otimes T'_d \). \( T'_d \) is the time reversal version of \( T_d \). The direct arrival between the receiver positions at the surface \( x_0 \) and the focusing point \( x_i \) can be estimated by finite-difference modeling and separated from the coda. Also, it can be computed in a direct way with an eikonal solver.

**Examples**

In this section, we will test the sensitivity to travel time in traditional Marchenko imaging. Figure 1 is the velocity model and the solid black curve is the travel time field of one shot. We use a 20 Hz Ricker wavelet to calculate the direct arrival and compute the reflection responses for 188 shots and 188 traces per shot. We carry out the tests with a velocity error \( \pm 8\% \) in the initial model respectively and use the smoothed versions to get travel time fields. The travel time fields from a single shot in different velocity models are given in Figure 2. Figure 3 is the comparisons of the direct arrivals from a specific focus point (1500, 1100) between different velocity models. Then we use the original data set and the different travel times calculated by eikonal solver to implement Marchenko imaging. The results are shown in Figure 4. Figure 4 (a) shows the conventional RTM image, with lots of artifacts. Figure 4 (c) is the image calculated using equation (4) based on the correct velocity model. Figure 4 (b) and (d) are the images calculated with the \( -8\% \) and \( +8\% \) velocity error travel time field. As we can see from Figure 4 (b), (c) and (d), all the results can image the subsurface structure roughly and clearly, despite Figure4 (b) and (d) using the incorrect travel times. Note that, due to the incorrect travel time fields, the images (b) and (d) seem like a shift up and down compared to the image (c). The dashed red curves indicate the true subsurface structures. Then we increase the velocity error up to \( +12\% \) and \( +16\% \), the results are shown in Figure 5 (a) and (b). With the error increases, the images shift more, and the ability to remove multiples reduced, more and more artifacts due to multiples appear.

*Figure 1* Velocity model. The solid black curve is the travel time field of one shot.
Figure 2 One single shot (shot number 50) travel time field from different velocity models.

Figure 3 Comparisons of the direct arrivals from a specific focus point (1500, 1100) between different velocity models. Red dashed lines indicate the same time.

Figure 4 Different imaging results. (a) is the conventional RTM image, having significant artifacts. (c) is the image calculated using equation (4) with the correct travel time field. Figure 4 (b) and (d) are the images calculated with the -8% and +8% velocity error travel time field. The dashed red curves represent the true subsurface structures. Red arrows indicate the locations of internal multiples.

Figure 5 Marchenko imaging with +12% (a) and +16% (b) velocity error travel time field. Compared with Figure 4 (d), as the velocity error increases, there are more and more artifacts due to multiples and the images shift more. Red arrows indicate the locations of internal multiples.
Conclusions

The Marchenko equations give us a method to calculate up- and downgoing Green’s functions. It takes care of downward continuation and yields better images than conventional RTM images. However, the traditional Marchenko imaging needs the knowledge of direct travel time information. The estimation of travel time has an impact on the results of imaging. When the estimation of direct arrival is not accurate, the retrieval of Green's functions will become inaccurate, and the ability to remove internal multiples become weaker, resulting in dislocations of reflectors and many artifacts due to incomplete cancelation of internal multiples.

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References


