Optimized acoustic wavefield modelling in transversely isotropic media

Introduction

Seismic imaging in anisotropic media is one of the most actual scientific research areas in seismology. The widely used anisotropic model is the transversely isotropic (TI) media with the vertical (VTI) or tilted (TTI) axis of symmetry. The application of reverse-time migration in such media is usually based on the acoustic TI approximation (Alkhalifah, 1998), avoiding shear waves' complications. However, the acoustic wave equation modelling usually suffers from shear wave artifact (Sabitov et al., 2017). Liu et al. (2009) proposed a pure quasi-P-wave equation described in the wavenumber domain. Usually, it is solved by simplifying the equation by various wavenumber approximations. Recently, Nikonenko and Charara (2020) proposed a way to formulate this task in a time-space domain for VTI media. We optimize this approach by stating the regression problem and extend the modelling for the TTI case.

Method

The pure quasi-P-wave equation in VTI media obtained by Liu et al. (2009) has the following form:

$$\left( \omega^2 - \frac{1}{2} V_p^2 \left( k_x^2 (1 + 2 \varepsilon) + k_z^2 + \sqrt{(k_x^2 (1 + 2 \varepsilon) + k_z^2)^2 + 8 k_x^2 k_z^2 (\delta - \varepsilon)} \right) \right) \tilde{P} = 0, \quad (1)$$

where $\tilde{P}(x, z, \omega)$ is the wavefield in the wavenumber domain, $\varepsilon$ and $\delta$ are the dimensionless Thomsen’s parameters (Thomsen, 1986), $V_p$ is the vertical P-wave velocity. Since it is impossible to solve this equation directly in a time-space domain in heterogeneous media because of the indivisible radical part, Nikonenko and Charara (2020) proposed to make an operator by the Fourier window method:

$$\tilde{\Lambda}(k_x, k_z, \varepsilon, \delta) = -\frac{1}{2} \left( k_x^2 (1 + 2 \varepsilon) + k_z^2 + \sqrt{(k_x^2 (1 + 2 \varepsilon) + k_z^2)^2 + 8 k_x^2 k_z^2 (\delta - \varepsilon)} \right). \quad (2)$$

Considering this operator $\tilde{\Lambda}(k_x, k_z, \varepsilon, \delta)$ as the desired wavenumber response, they apply inverse Fourier transform and cut the output by a 2D window to get the operator-mask $\Lambda_n(x, z)$ of any selected size. This operation is performed for all $N$ different pairs of anisotropic parameters $\varepsilon$ and $\delta$. Multiplication in the wavenumber domain corresponds to a convolution in a time-space domain. As a result, the task is reduced to the application of 2D convolution of a particular operator with the wavefield in all $N$ anisotropic areas:

$$\frac{\partial^2 P(x, z)}{\partial t^2} - \frac{1}{2} V_p^2 \left[ \Lambda_n(x, z) * P(x, z) \right] = 0, \quad n = 1, 2, ..., N. \quad (3)$$

This method is not optimal because the 2D window is not unique for the different problem settings. It is the same for any $\varepsilon$ and $\delta$ and leads to inaccuracy at strong anisotropy. We restate this task into finding a solution to the optimal problem and also extend the method for an application not only in VTI but also in TTI media. First, let us apply the rotation in the wavenumber domain to get the rotated operator 2:

$$\begin{align*}
\hat{k}_x &= k_x \cos \theta + k_z \sin \theta, \\
\hat{k}_z &= -k_x \sin \theta + k_z \cos \theta,
\end{align*} \quad (4)$$

where $\theta$ is the tilted angle, $\hat{k}_x$ and $\hat{k}_z$ are the rotated horizontal and vertical wavenumbers. As a result, we obtain the rotated operator:

$$\hat{\Lambda}(\hat{k}_x, \hat{k}_z, \varepsilon, \delta, \theta) = -\frac{1}{2} \left( \hat{k}_x^2 (1 + 2 \varepsilon) + \hat{k}_z^2 + \sqrt{(\hat{k}_x^2 (1 + 2 \varepsilon) + \hat{k}_z^2)^2 + 8 \hat{k}_x^2 \hat{k}_z^2 (\delta - \varepsilon)} \right). \quad (5)$$

The forward Fourier transform of the desired time-space domain operator $\Lambda_n(x, z)$ can be written as

$$A(\hat{k}_x, \hat{k}_z) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \Lambda_n(x, z) e^{-ik_x x} e^{-ik_z z} dxdz. \quad (6)$$
They can be determined by the following misfit function:

\[ F(\beta_{ij}) = \frac{1}{2} \sum_{k_x}^{N_x} \sum_{k_z}^{N_z} \left( -\frac{1}{2} \left( \hat{k}_x^2(1 + 2\epsilon) + \hat{k}_z^2 + \sqrt{(\hat{k}_x^2 + \hat{k}_z^2)} \right)^2 + 8\hat{k}_x\hat{k}_z(\delta - \epsilon) \right) \]

\[ - \sum_{i=-N_x/2}^{N_x/2} \sum_{j=-N_z/2}^{N_z/2} \beta_{ij} \left( \cos(k_x i\Delta x) \cos(k_z j\Delta z) - \sin(k_x i\Delta x) \sin(k_z j\Delta z) \right) \]

This linear regression problem can be solved using least-squares optimization. The coefficients \( \beta_{ij} \) depend on the point of the application, taking into account different Thomsen’s anisotropic parameters. They can be obtained before the entire modeling. The size of the final convolution mask, i.e., the number of coefficients, depends on the desired accuracy.
Numerical modelling examples

The numerical modelling is performed using a conventional finite-difference scheme and our optimized operator in a time-space domain. The conventional approach refers to the coupled differential equations of Duveneck and Bakker (2011). Nikonenko and Charara (2020) showed that the classical equation 1 is exactly the pure P-mode for that coupled equations. Figure 1 shows the modelling snapshots for the homogeneous problem. Figure 2 presents the 2-layered medium modelling as the ability of the operator to deal with the heterogeneous task. Figure 3 shows the parameters of the resampled BP 2007 TTI model. In total, 300×600 points are used. Figures 4 and 5 present modelling snapshots and corresponding seismograms for the BP TTI case. These examples reveal that our method successfully deals with complex models without generating the S-wave artifact while remaining accurate.

**Figure 1.** Modelling snapshots at 0.8 s by (a) the optimized TTI operator and (b) the conventional method for a 2D homogeneous acoustic anisotropic model. \( V_P = 2500 \text{ m/s}, \varepsilon = 0.2, \delta = 0.1, \theta = \pi/6, \tau = 0.001 \text{ s}, \Delta x = \Delta z = 10 \text{ m} \). The centrally located Ricker wavelet source function of 15 Hz is used.

**Figure 2.** Modelling snapshots at 0.8 s by (a) the optimized TTI operator and (b) the conventional method for a 2D 2-layered acoustic anisotropic model. \( V_P = 2500 \text{ m/s}, \varepsilon = 0.1, \delta = 0.1, \theta = \pi/6 \text{ in the first layer and } V_P = 3000 \text{ m/s}, \varepsilon = 0.15, \delta = 0.1, \theta = \pi/3 \text{ in the second layer. } \tau = 0.001 \text{ s}, \Delta x = \Delta z = 10 \text{ m} \). The centrally located Ricker wavelet source function of 15 Hz is used.

**Figure 3.** The BP 2007 TTI model for (a) \( V_P \), (b) \( \varepsilon \), (c) \( \delta \), and (d) \( \theta \).
Figure 4. Modelling snapshots at 1.4 s by (a) the optimized TTI operator and (b) the conventional method for the BP 2007 TTI model. \( \tau = 0.001 \) s, \( \Delta x = \Delta z = 30 \) m. A Ricker wavelet source function of 10 Hz is used, located at (9, 3.6) km.

Figure 5. Comparison of seismograms recorded at \( z = 3.6 \) km for (a) the optimized TTI operator and (b) the conventional method for the BP 2007 TTI model.

Conclusions

We have presented the new optimized approach in a time-space domain to solve the pure quasi-P-wave equation for TTI media without the decomposition of the original pseudodifferential operator. The method is local and can be combined with any conventional time-space method, e.g., finite differences. Our technique does not produce artifacts and does not increase the computational costs.

References


