The frequency domain viscoacoustic wave equation can be expressed in compact form as:

\[
\frac{\omega^2}{c^2} u(x, \omega) + \nabla^2 u(x, \omega) = f(x, \omega)
\]

(1)

where \(\omega\) is the angular frequency, \(u(x, \omega)\) denotes the pressure wavefield, and \(f(x, \omega)\) is the source function. The complex valued velocity, \(\tilde{v} = v \left[ 1 - \frac{1}{Q} \right]\) includes the effect of the intrinsic attenuation, with \(Q\) as the quality factor that describes the intrinsic attenuation in viscoacoustic media. The higher the \(Q\), the less intrinsic attenuation. Thus, \(v\) is the wave speed of compressional waves, and \(I = \sqrt{-1}\).

If we use \(m\) to represent the squared complex-valued slowness \(\frac{1}{v^2}\), and \(c = \frac{1}{\sqrt{1 - \frac{1}{Q}}}\), equation 1 can be written as:

\[
\omega^2 mcu(x, \omega) + \nabla^2 u(x, \omega) = f(x, \omega).
\]

(2)

The frequency domain viscoacoustic wave equation can be expressed in compact form as: \(L(m, c)u(x, \omega) = f(x, \omega)\). \(L(m, c) = \omega^2 mc + \nabla^2\) is the impedance matrix, which is also referred to as the modelling operator. We propose to use EWI to achieve multiparameter inversion in viscoacoustic media. The objective function of EWI is stated as:

\[
E(u_i, f_e) = \min_{u_i} \frac{1}{2} \sum_{i} \|d_i - Cu_i\|^2 + \frac{\alpha^2}{2} \|L_0 u_i - f_e\|^2.
\]

(3)

where, \(i\) is the source index, and \(d_i\) is the recorded data. \(C\) is an operator mapping the wavefield onto the sensor locations. \(\alpha^2\) is a weighting parameter balancing the priority between data fitting and satisfying the background wave equation. At the beginning of the inversion process, we start with a background modeling operator \(L_0(m_0, c_0) = \omega^2 m_0 c_0 + \nabla^2\), which corresponds to the background initial medium parameters: \(m_0\) and \(c_0\). In equation 3, the modified source function \(f_e\) starts with the original source.
function $f$ in the inversion process. In order to calculate the wavefield, which needs to simultaneously satisfy the data fitting objective and the wave equation, we minimize $E$ by solving $\nabla_a E = 0$. Thus, the wavefield satisfies the following linear equation:

$$
\begin{pmatrix}
\alpha \mathbf{L}_0 \\
\mathbf{c}
\end{pmatrix} u_i = \begin{pmatrix}
\alpha f_{ei} \\
\mathbf{d}_i
\end{pmatrix}.
$$

(4)

This background wavefield will include only single scattered energy from the missing perturbations, and such energy often dominates the wavefield, compared to multiscattering. The modified source function which satisfies the wave equation can be calculated using the background operator $\mathbf{L}_0$ as follows:

$$
f_{ei} = \mathbf{L}_0 u_i.
$$

(5)

In equation 5, $\mathbf{L}_0$ is updated as a consequence of updating the background models. If we perturb parameters $m$, $c$ by $\delta m$, $\delta c$, respectively, and according to the Born approximation, $m$ and $c$ would be given by:

$$
m = m_0 + \delta m, \quad c = c_0 + \delta c.
$$

(6)

We plug equation 6 into the modelling operator $\mathbf{L}(m, c) = \omega^2 mc + \mathbf{V}^2$, we get

$$
\mathbf{L}(m, c) = \omega^2 m c + \mathbf{V}^2 = \omega^2 (m_0 + \delta m)(c_0 + \delta c) + \mathbf{V}^2 = \mathbf{L}_0(m_0, c_0) + \omega^2 \delta m c_0 + \omega^2 \delta c m_0 + \omega^2 \delta c \delta m.
$$

(7)

According to the relationship between the background and true viscoacoustic modelling operators shown in equation 7, we can derive the relationship between the original and modified source function. The frequency-domain wave equation $Lu = f$ is equivalent to

$$
(L_0 + \omega^2 \delta m c_0 + \omega^2 m_0 \delta c + \omega^2 \delta m \delta c)u_i = f_i, \quad L_0 u_i = f_i - (\omega^2 \delta m c_0 + \omega^2 m_0 \delta c + \omega^2 \delta m \delta c)u_i = f_{ei}.
$$

(8)

Though the Born approximation could break down and result in cycle-skipping when the perturbations are large, EWI is able to overcome this problem by enhancing the data fitting (Alkhalifah and Song, 2019). If we ignore the second-order approximation term $\omega^2 \delta m \delta c u$, we finally get

$$
f_{ei} = f_i - (\omega^2 \delta m c_0 + \omega^2 m_0 \delta c)u_i.
$$

(9)

Equation 9 shows that the modified source function $f_e$ contains the squared slowness and the attenuation perturbations, and they will act as secondary sources to generate multiscattering information during the inner iterations between equations 4 and 6. In our implementation of viscoacoustic EWI, we invert for the velocity first, and the intrinsic attenuation second in a sequential matter. It is natural to update the velocity first, as velocity is the dominant parameter controlling the wave propagation. In this two-stage update scheme, when we update one target parameter, we assume the other parameter perturbation is zero. According to equation 9, the medium parameter perturbations $\delta m$ and $\delta c$ can be calculated using:

$$
\delta m = \sum_i \frac{u_i (f_i - f_{ei})}{\omega^2 (u_i c_0)(u_i c_0)^* + \lambda_1}, \quad \delta c = \sum_i \frac{u_i (f_i - f_{ei})}{\omega^2 (u_i m_0)(u_i m_0)^* + \lambda_2},
$$

(10)

where, $\lambda_1$ and $\lambda_2$ are small values to avoid dividing over zero. Usually, we use one percentage of the maximum values of the denominators in equations 10. After calculating the parameter perturbations $\delta m$ and $\delta c$, we can update the background models using:

$$
m_0 = m_0 + \delta m, \quad c_0 = c_0 + \delta c.
$$

(11)

When convergence is achieved, the velocity and quality factor model are evaluated using:

$$
v = \Re \sqrt{\frac{1}{m_0}}, \quad Q = \Re \frac{i}{2(1 - \sqrt{1/c_0})}.
$$

(12)

The velocity and the quality factor $Q$ both have isotropic radiation patterns, and $Q$ has a lower sensitivity than the velocity to the data for all scattering angles. Thus, if we start the $Q$ inversion with a poor initial velocity model, the velocity error will affect the $Q$ inversion result. As parameter perturbations include multi-scattering components, the convergence rate in each parameter inversion accelerates (Alkhalifah and Wu, 2016; Song et al., 2019).
Examples

We apply the proposed method on a viscoacoustic Marmousi model. The true velocity model is shown in Figure 1a, and we Gaussian smooth the true model three times with a window size of 1000 x 1000 m to obtain the initial velocity, as shown in Figure 1b. The size of the model is 461 x 284, and the spatial sampling interval in both vertical and horizontal directions is 25 m. The true Q model is shown in Figure 1c, and the initial Q model is homogeneous with a high value of 100, as shown in Figure 1d.

![Figure 1](https://example.com/figure1.png)

Figure 1 The true (a), initial (b) Marmousi velocity models; the true (c), and initial (d) Marmousi Q models.

We use 23 sources uniformly distributed on the surface, and their locations are given by * symbols in Figure 1a. All the grid points on the surface act as receivers. We implement the proposed method in the frequency domain, and the selected frequency band extends from 3 Hz to 10 Hz with the sampling interval of 0.5 Hz. The synthetic data are generated from a point source function for all the selected frequencies using the true velocity and Q models. We first use an acoustic FWI and an acoustic EWI to invert for the velocity. After 50 outer iterations, the inverted velocity models are shown in Figures 2a and 2b, respectively. As the intrinsic attenuation effect is ignored using a pure acoustic approximation, FWI cannot recover well the detailed structures of the true model, especially in the deep area. Although acoustic EWI improves the inversion result a lot, the regions where the high intrinsic attenuation exists are still not well inverted, as the arrow points out. Next, we perform the sequential viscoacoustic FWI to invert for the velocity and Q models with the same inversion setup. The inversion results are shown in Figures 2c and 2d, respectively. The inverted velocity using sequential viscoacoustic FWI improves, but the details are not well recovered. The sequential viscoacoustic FWI inverted Q model is shown in Figure 3a, and it clearly results in the velocity update leakage. We then use viscoacoustic EWI to invert for the velocity with the fixed Q model in Figure 1d. After the same number of outer iterations, the inverted velocity model is shown in Figure 2d. Even with the fixed background Q, viscoacoustic EWI still improves the velocity inversion. Only a small part where high attenuation exists is not well recovered, as the arrow points out. Finally, we perform joint and sequential viscoacoustic EWI with the same inversion setup. The inverted velocity and Q models from joint viscoacoustic EWI are shown in Figures 2e and 3b, respectively. Figure 3b shows wrong intrinsic attenuation updates in the shallow area. By comparison, sequential viscoacoustic EWI is able to recover reasonable velocity and Q models, as shown in Figures 2f and 3c, respectively. All the inverted Q models include TV denoising to remove the high-wavenumber oscillations (Vogel and Oman, 1996).

Conclusions

We use an efficient wavefield inversion to achieve multiparameter estimation in viscoacoustic media. We invert for the velocity and the intrinsic attenuation parameter in a sequential matter to reduce the trade-off between the parameters. As the parameter perturbations are calculated using direct division, it doesn’t require a preconditioning to the gradient or a line search for the step length. So the computational cost is affordable. A test on a model with distinct Gaussian anomalies for the velocity and the intrinsic attenuation parameter shows that a sequential EWI can mitigate the trade-off problem effec-
Figure 2 The inverted velocity models using acoustic FWI (a), acoustic EWI (b), sequential viscoacoustic FWI (c), viscoacoustic EWI with fixed background Q (d), joint viscoacoustic EWI (e), and sequential viscoacoustic EWI (f).

Figure 3 The inverted Q models using sequential viscoacoustic FWI (a), joint viscoacoustic EWI (b), and sequential viscoacoustic EWI (c).

...tively. Application on synthetic data generated from an intrinsic attenuation Marmousi model shows that EWI can reasonably recover the viscoacoustic parameters.

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References


