Using a broadband wavelet for full waveform inversion on reflected seismic waves

Introduction

The conventional full waveform inversion (FWI) (Tarantola, 1984) could be used to invert both the low and high wavenumber components of the seismic velocity in exploration geophysics. To overcome the cycle-skipping problem resulted from the lack of accurate initial velocity model and low frequency components in the field seismic data, the FWI gradient is usually decomposed into tomography and reflection parts. For example, Xu et al. (2012) successfully use the reflected seismic waves to invert the macro velocity based on double born approximation to derive the tomography gradient. In this abstract, we assume the initial velocity is accurate and we try to improve the accuracy and resolution of the FWI result based on the reflection gradient. Two preconditions are introduced, the first one is to apply double integration on the data residual to keep both the high and low frequency components in the FWI gradient, and the other one is to use a broadband wavelet to replace the traditional wavelet to calculate the forward modeling data. Besides, we also apply the wavefield decomposition technology (Fei et al. 2015) in the imaging condition to remove the tomography gradient.

Method

The cost function for conventional FWI is defined as $C = (\Delta d^T \Delta d)/2$, where the data residual $\Delta d$ is defined as the difference between the calculated synthetic data $d_{cal}$ and the observed data $d_{obs}$. Based on the adjoint-state method, the FWI gradient can be formulated as

$$J = \frac{\partial C}{\partial s} = \left(\frac{\partial d_{cal}}{\partial s}\right)^T \Delta d = - d_{cal}^T \frac{\partial L}{\partial s} (L^* \Delta d). \quad (1)$$

In equation 1, $J$ is the FWI gradient, $L$ is the forward modelling operator and $L^*$ is the adjoint of $L$ (Liu et al. 2019), which is used as the backward modelling operator. $d_{cal}^T$ is the forward modelled data, $L^* \Delta d$ is the back propagated data residual, and the operator $\partial L/\partial s$ is proportional to $\omega^2$ in the frequency domain or second order derivatives in the time domain. The FWI gradient in equation 1 can also be written as an zero-lag cross-correlation imaging condition as

$$J(x, z) \sim \int_0^{T_{\text{max}}} S(t, x, z) \frac{\partial^2}{\partial t^2} R(t, x, z) dt \sim \int_0^{\omega_{\text{max}}} -\omega^2 S(\omega, x, z) R^*(\omega, x, z) d\omega. \quad (2)$$

In this equation, $S(t, x, z)$ is the time-domain forward modelled data and $S(\omega, x, z)$ is the equivalent in frequency domain, $R(t, x, z)$ and is the time-domain back propagated data residual and $R^*(\omega, x, z)$ is the equivalent in frequency domain. From equation 2, we observe that the data residual is filtered by both $\omega^2$ and $S(\omega, x, z)$. During our study, we find that these filters will degrade the inversion results. To solve this problem, we apply two preconditions during FWI. The first precondition is to apply double integration on the data residual to remove the effect of $\omega^2$, and the second precondition is to utilize a broadband wavelet to calculate $S(t, x, z)$.

The blue solid line in Figure 1a denotes a 10 Hz Ricker wavelet, the red line denotes a broadband wavelet, and their spectral are denoted in Figure 1b. Treating these two wavelets as two filters and applying them on the 10 Hz Ricker wavelet in Figure 1a, we can get two filtered signals in time domain as shown Figure 1c or in frequency domain as shown in Figure 1d. From the comparison, the signal filtered by the broadband wavelet does not change in both time and frequency domains, while the one filtered by the Ricker wavelet evolves new side lobes in time domain and both the high and low frequency components are suppressed in the frequency domain, and these effects will decrease the accuracy of the final FWI result.

For the reflected seismic waves, the FWI gradient can be decomposed into tomography and reflection parts. In our method, we mainly focus on using the reflection gradient for inversion, so we apply the following “de-primary” (Fei et al, 2015) imaging condition to remove the tomography gradient.
In this equation, \( h_z \) means Hilbert transform along the vertical direction, by changing the subtraction sign to addition, the equation can be used to obtain the tomography gradient. We will compare the different inversion results in the next section.

\[
J(x, z) = \int_0^{T_{\text{max}}} (SR - h_z(S)h_z(R))dt .
\] (3)

**Examples**

We use a 1.5D model indicated by the red line in Figure 4 to test the preconditions for FWI. A 10 Hz wavelet is applied to generate a synthetic data shown in Figure 2a. As a comparison, synthetic data using broadband wavelet is shown in Figure 2b. When calculating the data residual, we need to estimate a wavelet by applying Wiener filter between the two datasets. The estimated wavelet convolves Figure 2b to obtain the calculated data \( d_{\text{cal}} \) in equation 1.

**Figure 1** Broadband wavelet. (a) and (b) indicate the traditional Ricker wavelet (blue line) and broadband wavelet (blue line) in time and frequency domains, respectively. (c) and (d) indicate the filtered Ricker wavelet by a broadband (red) or a Ricker wavelet (blue) in time and frequency domains.

**Figure 2** Synthetic data generated using Ricker in (a) and broadband wavelet in (b).

We start from a constant velocity (black line in Figure 4) and apply full traveltime inversion (FTI, Luo et al, 2016) to invert a smooth initial velocity (green line in Figure 4) for FWI. Figure 3a indicates the conventional FWI gradient, Figures 3b is the gradient after applying the double integration precondition, and Figure 3c is the gradient after applying the broadband wavelet for forward modelling. Figure 4
shows the inversion results using the three gradients after 20 iterations, and Figure 5a compares the convergence history of the three methods. By converting the velocities from depth to time domain, we can calculate the frequency spectral of the different inversion results in Figure 5b and the FWI result after applying the two-step preconditions matches the true solution better than the other algorithms.

**Figure 3** FWI gradients of conventional FWI in (a), precondition one in (b), and preconditions one and two in (c).

**Figure 4** Inversion results of conventional FWI in (a), precondition one in (b), and preconditions one and two in (c).

**Figure 5** Convergence history in (a) and spectral of inversion results in (b).

In the previous tests, we only consider the reflection gradient for the inversion. We will now compare the inversion results using tomography and reflection gradients. Figure 6a shows the tomography gradient, Figure 6b indicates the conventional FWI gradient without decomposition, and Figure 6c (same as Figure 3c) is the reflection gradient. To keep consistency, the two-step preconditions are all applied for these gradients. Starting from the FTI velocity (green lines in Figure 7), the inversion results using the three gradients are shown in Figure 7. From Figure 7a, the FWI result using the tomography gradient is very similar to the FTI result, which means it cannot recover the high resolution details, the inversion result from the reflection gradient (Figure 7c) matches nearly perfect with the true solution and the resolution is much better than the conventional FWI result shown in Figure 7b.
Conclusions

We introduce two-step preconditions for FWI on reflected seismic waves, the first step is to apply double integration on the data residuals and the second step is to use a broadband wavelet to do forward modelling. Numerical tests prove that these preconditions can better preserve the frequency content of the original seismic data, and so the inversion is more robust than the conventional FWI. Besides, we also compare the inversion using the tomography and reflection gradients and conclude that to get a higher resolution inversion, the tomography gradient needs to be removed during the inversion. Except for the seismic exploration, the introduced technology is also being applied for medical imaging using ultrasonic waves.

Acknowledgements

The author wants to thank Yi Luo and Houzhu Zhang for the discussion of this work.

References