Introduction

Deconvolution is one of the main processing algorithms for removing the wavelet and improving the temporal resolution of seismic data. Earth is an LTI system; hence, the convolution theorem is valid. The recorded seismic data is the convolution of the subsurface reflectivity series with the wavelet. A class of deconvolution algorithms that implements sparsity constraint to recover a sparse reflectivity series from the seismic recordings called sparse spike deconvolution (Levy and Fullagar, 1981; Velis, 2007; Duval and Peyré, 2015). There are also blind multichannel algorithms that aim at recovering a sparse reflectivity series without having information about the wavelet (Kazemi and Sacchi, 2014; Kazemi et al., 2016). These algorithms are extensively studied and used in the industry. Unfortunately, the performance of these algorithms can be deteriorated by a moderate amount of noise in the data. Hence, in the presence of noise, sparsity assumption alone is not enough and the deconvolution is unstable (Kazemi, 2018). Researchers are actively looking for new approaches to increase the noise tolerance behavior of the sparsity-based deconvolution algorithms. Noise has a significant impact on the phase than the amplitude spectrum. In other words, a moderate amount of noise can drastically change the phase information in the data. In many fields of study, such as X-ray crystallography, microscopy, astronomy, diffraction imaging, and optics the phase spectrum is either too difficult to measure or it is too noisy (Qu et al., 2019). Hence, researchers are developing algorithms such that they only require amplitude information to retrieve the signal of interest. Previous attempts show that in specific circumstances, the phase retrieval is possible.

Recently, Qu et al. (2019) propose a convolutional phase retrieval based on local optimization that aims at recovering the signal of interest from only measuring the amplitude information of the convolution of the signal with a random operator. They show that if the measurements have a high signal to noise ratio (SNR) and the convolution matrix is a random operator, the method can accurately recover the signal. However, in seismic recordings, the convolution matrix is not a random operator. It is built from the wavelet. Hence, this method cannot be directly applied to seismic deconvolution. Also, in seismic we do have access to the phase information even though it is corrupted by noise. Nonetheless, as we mentioned earlier the amplitude spectrum is more resilient to noise and the phase retrieval method can be used as a strong constraint to reduce the model space and improve the efficiency of sparse spike deconvolution algorithms. Accordingly, in this paper, we develop a hybrid algorithm that uses the phase retrieval constraint along with the sparsity assumption on reflectivity series to handle problems with low SNR measurements. Our phase retrieval sparse spike deconvolution algorithm is a local minimization technique and provides reliable solutions when the data is too noisy, i.e., $SNR \geq 2$.

Theory

The seismic trace can be written as

$$ d = W r + n, \quad (1) $$

where $r = (r[0], r[1], \ldots, r[M - 1])^T$ is reflectivity series, $W$ is a convolution matrix of wavelet built from the seismic source signature $w = (w[0], w[1], \ldots, w[L - 1])^T$, $n$ is noise, and $d = (d[0], d[1], \ldots, d[N - 1])^T$ is the recorded seismic trace. Note that $N = M + L - 1$, and $W$ with dimensions $N \times M$ is

$$ W = \begin{bmatrix} w(0) & w(1) & w(0) \\ w(1) & w(0) & \vdots \\ w(2) & w(1) & \ddots \\ \vdots & \ddots & \ddots \\ w(L - 2) & w(L - 1) & w(L - 1) \\ w(L - 1) & w(L - 1) & \end{bmatrix}. \quad (2) $$

Given the source wavelet, sparse spike deconvolution algorithms aim at estimating the sparse reflectivity series by minimizing

$$ \{ \hat{r} \} = \operatorname{argmin}_{r} J(r) = \| W r - d \|_2^2 + \lambda \| x \|_1, \quad (3) $$

where $\lambda$ is a regularization parameter and $\ell_1$ norm is used as a regularization term to promote sparsity in the reflectivity series. Equation (3) is a linear optimization problem and any $\ell_2 - \ell_1$ solver can be used.
to minimize the cost function efficiently. To take advantage of the gradient type algorithms, we replace the non-smooth $\ell_1$ norm term with a hybrid norm that is differentiable

$$\{\hat{r}\} = \arg\min_r J(r) = \|Wr - d\|_2^2 + \lambda \sum_i (\sqrt{r_i^2 + \epsilon^2} - \epsilon), \quad (4)$$

when $\epsilon$ is a small number. To better approximate the $\ell_1$ norm with the hybrid penalty function, the $\epsilon$ should be chosen as a small positive number close to zero. The cost function of Equation (4) is differentiable and the gradient type algorithms can be used to solve the problem. However, Equations (3) and (4) struggle when the recorded data is contaminated with strong noise. In other words, in the presence of noise fitting the data in the $\ell_1$ norm sense while searching for a sparse solution does not provide acceptable results.

This observation motivated us to provide a stronger constraint into the problem. The noise term drastically corrupts the phase spectrum of the data but the SNR in the amplitude spectrum, especially in the bandwidth defined by the source signature, is higher than the time and phase-only domains. Hence, a phase retrieval constraint is added to the cost function of Equation (3) to make the algorithm stable in the presence of noise. We propose to solve

$$\{\hat{r}\} = \arg\min_r J(r) = \|Wr - d\|_2^2 + \beta \|b^\dagger \odot (|Ar| - y)\|_2^2 + \lambda \sum_i (\sqrt{r_i^2 + \epsilon^2} - \epsilon), \quad (5)$$

where $A = FW$, $F$ is forward Fourier transform operator, $|v|$ is the absolute value of the function $v$, $\odot$ is Hadamard product operator, $b$ is a weighting function, $y = [Fd]$ is the amplitude spectrum of the data $d$. The regularization parameter $\beta$ balances the importance of data fitting in the time domain and amplitude matching in the frequency domain. The cost function of Equation (5) is nonlinear and due to the amplitude matching term, i.e., the second term in the cost function, the cost function is non-convex and non-smooth. However, similar to the method of Wang et al. (2017), the local optimization of Equation (5) via generalized gradient descent is possible. The generalized gradient of the cost function of Equation (5) reads

$$\frac{\partial J(r)}{\partial r} = W^T(Wr - d) + \beta A^* diag(b)(Ar - y \odot \exp(i\phi(Ar))) + \lambda \frac{r}{r^2 + \epsilon^2}, \quad (6)$$

where $i = \sqrt{-1}$, $*$ stands for conjugate transpose, $diag(v)$ operator shapes the vector $v$ to a diagonal matrix, and $\phi(v)$ is the phase of $v$. Then, gradient descent update rule is

$$r^{k+1} = r^k - \alpha \frac{\partial J(r^k)}{\partial r}, \quad (7)$$

where $k$ is the iteration number, and $\alpha$ is the step size. To avoid local minima in local optimization algorithms, the initial solution is of great importance. Hence, similar to the works of Qu et al. (2019), we implement a careful initialization by using the spectral method. The initial solution is computed as $r^0 = \tau \hat{r}^0$, where $\tau = \|y\|_2$, and $\hat{r}^0$ is the leading eigenvector of $A^* diag(y^2)A$ matrix. This initialization step guarantees the convergence of the algorithm to the desired solution. To speed up the inversion process, we can adopt non-linear optimization algorithms. Here, we use the L-BFGS method to solve the cost function of Equation (5).

**Examples**

To evaluate the performance of the method, we use a noise-free synthetic data. We generated 50 reflectivity coefficients with random amplitudes (Figure 1b). Then, the reflectivity series is convolved with a 40-degree phase rotated Ricker wavelet to generate the data (Figure 1a). Figure 1c shows the estimated reflectivity series after the application of the proposed algorithm. For all of the examples, we initialize the algorithm with the spectral initialization method. Through trial and error, we choose $\beta = 0.0001$, and $\lambda = 0.001$. Figure (1c) shows that the algorithm successfully recovers the reflectivity series.
Figure 1 Application of the method on a clean synthetic data with SNR=100 and its amplitude spectra. a) Original data, b) true data, c) estimated reflectivity series, and d) band-pass filtered reflectivity series.

Figure 2 Application of the method on a noisy synthetic data with SNR=2 and its amplitude spectra. a) Original data, b) true data, c) estimated reflectivity series, and d) band-pass filtered reflectivity series.

use two figures of merits similar to the work of Kazemi and Sacchi (2014), namely the quality of reconstruction $Q$, and the normalized correlation coefficient $NCC$. We obtain a reconstruction quality of $Q = 39$ (dB), and a normalized correlation coefficient $NCC = 0.99$ for the reflectivity series. To test the algorithm in the presence of noise, we rerun the algorithm with additive noise $SNR = 2$ (Figure 2). A band-limited Gaussian noise is added to the noise-free data in Figure 1a to generate the noisy dataset represented in Figure 2. In this case, we set $\beta = 0.0001$, and $\lambda = 0.18$. Results in Figure 2 indicate that the algorithm is successful in recovering the main energy of reflectivity series, however, there are fictitious coefficients too. Compared to the existing algorithms, our method provides stable results and is less sensitive to noise. We achieve the quality of reconstruction of $Q = 8.9$ (dB) and a normalized correlation coefficient of $NCC = 0.87$ for the reflectivity series. Next, we apply the algorithm on lines A and D of the Teapot Dome datasets. Our algorithm assumes that we have access to the source wavelet, and only solves for the reflectivity series. Hence, we provide the wavelet to the algorithm. The wavelet is borrowed from the works of Kazemi (2018). On other datasets, the source signature can be estimated.

Figure 3 Application of the proposed algorithm on the line A of Teapot Dome dataset. a) Line A of Teapot Dome dataset. b) Estimated reflectivity series.
Figure 4 Application of the proposed algorithm on the line A of Teapot Dome dataset. a) Line A of Teapot Dome dataset. b) Estimated reflectivity series.

from well-log information. After providing the wavelet, our method is successfully applied on both lines and provided the high-resolution reflectivity sections. Figures (3) and (4) shows the results on line A and D, respectively. In both cases, we choose $\beta = 0.0001$, and $\lambda = 0.3$.

Conclusions

We have developed a noise-tolerant sparse spike algorithm. We show that in the presence of noise the amplitude spectrum is less corrupted. On the other hand, moderate levels of noise can drastically degrade the phase of data. Hence, we proposed to use the amplitude matching as a regularization term to deliver a noise-tolerant algorithm. The new cost function based on data matching in the time and amplitude domains along with sparsity constraint on the reflectivity series resulted in non-linear optimization. To solve this problem, we use a local optimization method with a careful initialization step. Here, we used the L-BFGS method with spectral initialization. Our results on synthetic and real data examples show that the method can tolerate noise $\text{SNR} \geq 2$ and provide reliable reflectivity series.

References


