Direct Bayesian seismic inversion for porosity estimation in a hard rock carbonate reservoir

Introduction
The carbonate reservoir under study is located in southwest Iran. The approximately 300-m thick reservoir is divided into thirteen subzones (Figure 1). The petrophysical analyses show that the reservoir formation is comprised of dominantly calcite mineral with a small amount of shale (6%). Porosity varies between 4%-25% and is the key control on the reservoir quality. With an average porosity of 16%, the most prolific intervals are Sar3, Sar8, Sar12. Characterisation of the reservoir porosity and its uncertainty is the key component of reservoir development strategies using the seismic data. From a mathematical point of view, this procedure is an ill-posed non-linear inverse problem, which is cast into a Bayesian framework. In contrast to the deterministic inversion methods, the probabilistic approaches can explore the full space of uncertainty of the model parameters and account for complex prior information reliably (Tarantola and Valette, 1982). The solution is a posterior probability density function (PDF), which expresses the probability of model occurrence given the observed data. The capability of statistical analyses of the posterior PDF such as assessment of the uncertainty of the model realisations made the probabilistic inversion approaches a powerful means for reservoir properties description (Bosch et al. 2010). For direct inversion of seismic data to porosity, the rock-physics model (RPM) was integrated into the inversion workflow. In addition to reservoir porosity and its uncertainty, facies probability and variable critical porosity volumes were derived.

Rock-physics model calibration and seismic well-tie
The available compressional and sonic and density logs, as well as petrophysical logs (porosity, the volume of shale, water saturation, and gamma-ray) from four wells, were used to calibrate Nur’s critical porosity model (Nur et al. 1998). Here, we developed the methodology proposed by Heidari et al. (2020) and used a facies-dependent variable critical porosity model. The match of the observed and modelled elastic moduli and density logs are shown in Figure 1. The well-to-seismic tie procedure was performed using a statistical wavelet (with the dominant frequency of 24 Hz) to obtain the appropriate scaler to be used in the seismic inversion. The seismic well-ties are shown on the seismic section passing through the four well locations. The tops of the reservoir subzones (the black dashed lines), the location of the wells (the vertical dashed lines), and their approximate distance, as well as the synthetic seismic traces estimated through the well-to-seismic analysis, are superimposed on Figure 2a.

Parameterisation of the probabilistic inversion
Probabilistic seismic inversion was performed using a Markov chain Monte Carlo sampling-based method via the Extended Metropolis-Hastings algorithm (Mosegaard and Tarantola, 1995; Hansen et al., 2014). Based on the statistical analysis of the available porosity logs, we assumed that the prior model could be represented by a multivariate Gaussian distribution. Fast Fourier Transform Moving Average (FFTMA, Le Ravalec et al. 2000) method was used to represent the prior distribution of the porosity logs. Our inversion feasibility study based on synthetic seismic data indicates that noise parameterisation is an important aspect of the inversion setup. However, the characterisation of the noise in the real seismic data is not straightforward. Therefore, three different methods, I) the residuals from seismic well ties, II) residuals from an existing seismic inversion, and III) a curvelet-based denoise filtering were used to gain insight into noise in our seismic data. As expected, the noise magnitude from these methods varies and the signal-to-noise ratio (SNR) were 1.1, 1.6, and 3.3 respectively. Our feasibility analysis showed that while underestimating the noise could result in the erroneous estimation of the porosity uncertainty, its overestimation results in a smoother mean of the posterior realisations. Therefore, we examined the results of the inversion with noise magnitude obtained from the three approaches above. The inversion results indicate that the noise from cases I and II produces vertical stripes on the posterior mean (not shown here), which are attributed to the convergence issues as the sampling algorithm does not reach to burn-in phase and results in overfitting the data. Here, only the results for the noise magnitude (SNR = 3.3) are shown. In contrast to the routine practice of using uncorrelated white noise in probabilistic seismic inversion studies, to account for the vertical coupling of noise samples, we assumed a correlated Gaussian noise model, where the covariance matrix was constructed based on the extracted seismic wavelet. The algorithm was run trace by trace for 400000 iterations to ensure that the full posterior distribution is sampled within a reasonable computation time.
Results and discussions

We investigated the performance of the inversion by analysing the statistics of the porosity posterior realisations for the seismic section and well logs. The posterior mean of the porosity realisations is shown in Figure 2b. This figure demonstrates that the inversion could resolve some thin layers (see white arrows). Besides, some features on the seismic section, related to disconformities are successfully resolved by inversion (black ellipses). Using the optimized regression obtained through the rock-physics calibration procedure, the mean porosity model was converted to critical porosity (Figure 1c). In conjunction with core and thin sections analysis, the critical porosity volume could be used to assess the diagenetic evolution, pore-network structure, and elastic properties of the reservoir (Fournier and Borgomano, 2009). In contrast to the porosity posterior mean, the variations of the critical porosity are more pronounced. For example, the low porosity region between the two horizons flagged by the black arrows is characterized on its associated critical porosity by resolved thin layers with variable critical porosity (see red ellipses on Figures 2b and c). Using the porosity posterior mean model, it is not trivial to distinguish the regions with porosity variations within a specific range. In order to evaluate the uncertainty of the posterior realisations, we obtained the probability values of the posterior mean for two facies. Figures 2d and 2e depict the probability values associated with facies-1 with the porosity between zero and 15%, and facies-2 with the porosity between 15% and 30%, respectively. These probability sections facilitate the interpretation of the posterior mean model as they indicate the uncertainty associated with the inverted porosities. In addition, the interpreter can easily flag the regions where the porosity values are in the desired range. The statistics of the porosity posterior realisations, i.e. their mean, the 95% confidence interval (CI), as well as the measured porosity logs are shown for four wells (Figure 3). Considering the imperfect well-ties, the posterior mean model has a reasonable match to the measured porosity logs for the four well, except in zones flagged by yellow arrows. The mismatch between the predicted and observed porosity model is more notable in well W06n. This resulted in the failure of capturing the trend of the true porosity even in relatively thicker Sar12 interval in wells W06n and W10. In addition to well-tie issues, the inconsistency between the inverted and observed porosity could also be attributed to inversion inability to resolve the thin layers as well as the error related to the fact that our rock-physics model does not include the shale lithology. For example, the inversion could not resolve the thin shale layer at Sar2 interval and overestimates its porosity.

![Figure 1](image_url)
Figure 2  a) The seismic section superimposed by the synthetic traces obtained through the well-to-seismic tie, and the top of the reservoir sub-zones, and the extracted statistical seismic wavelet, as well as the well locations, b) the mean of the porosity posterior realisations, c) the critical porosity model, d) the probability of the porosities between zero and 15% (facies-1), e) the probability of the porosities between 15% and 30% (facies-2).
Figure 3 The statistical features of the porosity realisations at four well-locations. For each well log: The colour-coded column shows the reservoir tops. The porosity realisations are depicted as the density plots, where the background colours represent the probability of the realisations at each time sample. The posterior mean, the observed porosity, and the 95% CIs are demonstrated with red, black, and dashed pink lines, respectively. The results of the well-to-seismic tie are shown in the two last columns.

Conclusions
This case study demonstrates the applicability of the probabilistic seismic inversion to characterize the porosity directly from seismic data. The performance of the inversion in resolving the thin layers and quantifying the uncertainty of the porosity posterior realisations through the probability sections confirms the suitability of our assumptions in creating the prior information as well as noise model. No overfitting of the data and no unrealistic artificial features were observed on the posterior mean sections. Moreover, more than 87% of the observed porosity samples are within a 95% confidence interval, which all indicate that our assumptions about the noise model as well as the prior information were appropriate.

References