A method to predict S-wave velocity from wire-line logs for organic-rich shales

Introduction

S-wave velocity is a key parameter in the identification of lithology and fluid in reservoirs because it provides subsurface shear properties relating to rock types and fluid saturation. Therefore, pre-stack seismic inversion, brittleness index calculation, stress analysis, reservoir prediction and formation evaluation require knowledge of S-wave velocity (Wang et al., 2019). S-wave velocity data are commonly collected using dipole shear wave logging. However, these logging tools are not implemented routinely (Xu and White, 1995). There is a demand to estimate S-wave velocity from wire-line logs whenever S-wave velocity logs are unavailable. Recent studies on rock physics shear wave velocity prediction methods for sandstone (Xu and White, 1995; Liu et al., 2015) and carbonate (Xu and Payne, 2009; Sun et al., 2012) reservoirs are relatively mature, however, studies on organic-rich rocks are few. Organic-rich rocks differ from sandstone and carbonate reservoirs in that they contain a certain amount of organic matter.

Based on the research of Hou et al. (2012) and Tan et al. (2015), this paper proposed a S-wave velocity prediction method for organic-rich rock. This method takes the effect of organic matter on the velocity into consideration. Meanwhile, this method utilized critical porosity and consolidation coefficient to characterize the effect of pore structure, consolidation degree and diagenetic evolution history. The critical porosity and consolidation coefficient are variable, which can be obtained by using the P-wave velocity. Then, the S-wave velocity can be calculated by introducing these variable parameters into the rock physics model. The critical porosity and consolidation coefficient obtained from each sample point can improve the description accuracy of the rock physics model for organic rich rocks and thus reduce the shear wave velocity prediction error.

Method and theory

• Calculation of velocity

Given the dry rock elastic moduli and utilized the Gassmann equation, the bulk and shear modulus of fluid-saturated isotropic rock at low frequency are obtained (Gassmann, 1951):

\[
\frac{K_s}{K_m - K_s} = \frac{K_d}{K_m - K_d} + \frac{K_f}{\phi(K_m - K_f)}
\]

(1)

\[
G_s = G_d
\]

(2)

Where, \(K_s\), \(K_d\), \(K_m\), \(K_f\) are the bulk moduli of the fluid saturated rock, dry rock skeleton, rock matrix and fluid in pores, respectively. \(G_s\) and \(G_d\) denote the shear moduli of the fluid saturated rock and dry rock skeleton. \(\phi\) is the rock total porosity.

The P- & S-wave velocity of saturated rock can be calculated with the following equations,

\[
V_p = \sqrt{\frac{K_s + 4/3G_s}{\rho_s}}
\]

(3)

\[
V_s = \frac{G_s}{\sqrt{\rho_s}}
\]

(4)

Where, \(V_p\) and \(V_s\) are the P- and S-wave velocity of saturated rock, \(\rho_s\) is the density of the saturated rock.

• Elastic moduli of dry rock frame

Hou et al. (2011) derived the critical-porosity consolidation coefficient (CPCC) model through introducing critical porosity (CP) into the consolidation coefficient (CC) model.

\[
K_d = \frac{K_m(1-\phi/\phi_c)}{1+\alpha\phi/\phi_c}
\]

(5)
\[ G_d = \frac{G_m(1-\varphi/\varphi_c)}{1+\gamma \alpha \varphi/\varphi_c} \]  

Where, the rock total porosity is subject to 0<\varphi<\varphi_c, so the range of \( \varphi_c \) is from \( \varphi \) to 1, the consolidation coefficient ranges between 0 and 20. This model extends the CP model and CC model. It can be seen from equation (5) and equation (6), when \( \alpha = 0 \), equation (5) and (6) are degenerated to CP model. When \( \varphi_c = 1 \), equation (5) and (6) are degenerated to CC model.

Hou et al. (2011) proposed a procedure to predict S-wave velocity of saturated rock using the CPCC model. In their procedure, the critical porosity \( \varphi_c \) is set as a constant value. S-wave velocity is predicted through optimizing the consolidation coefficient \( \alpha \) and minimizing the difference between the predicted P-wave velocity and measured P-wave velocity. Essentially, the advantage of the CPCC model is not fully exerted since this method does not calculate the specific values of the critical porosity and consolidation coefficient at the same time. Figure 1 and 2 show the P- and S-wave velocities vary with \( \varphi_c \) and \( \alpha \).

**Figure 1. Diagram indicating the P-wave velocities vary with \( \varphi_c \) and \( \alpha \).**

**Figure 2. Diagram indicating the S-wave velocities vary with \( \varphi_c \) and \( \alpha \).**

- S-wave velocity prediction strategy

This paper proposes a S-wave velocity prediction method for organic rich rocks based on the integration of CPCC rock physics model with variable parameters, VRH average equation, Wood equation and Simulated Annealing(SA) optimization algorithm. This method takes the effect of organic matter on the velocity into consideration. The critical porosity and consolidation coefficient are variable, which can be obtained by using the P-wave velocity. Then, the S-wave velocity can be calculated by introducing these variable parameters into the rock physics model. Figure 3 shows the workflow to predict shear wave velocity based on the critical porosity and consolidation coefficient inversion. The procedure is as follows: (1) This paper get the volume of minerals and kerogen, water saturation, porosity and P-wave velocity from conventional logging data or lab testing data. The elastic moduli and density of minerals, kerogen and fluids are obtained from scholar’s literature. (2) The VRH average equation is used to calculate the bulk and shear moduli of the rock matrix. It is suggested that the kerogen is considered as a special “mineral”, a part of rock matrix. (3) The Wood equation(1955) is applied to estimate the bulk modulus of mixed fluids \( K_s \). (4) The CPCC model is utilized to calculate the dry rock skeleton moduli. In this step, the consolidation coefficient and critical porosity are variable at the same time. (5) The bulk and shear moduli of saturated rock can be calculated by the Gassmann’s equation (equation 2.1 and 2.2). (6) The P- and S-wave velocity of fluid-
saturated rock is calculated by the equation (3) and (4). Then, the relation between the P-wave velocity \(V_c\) and the variable parameters \((\phi_c, \alpha)\), \(V_c = f(\alpha, \phi_c)\) is established. According to the feature of this relation, the two parameters \((\alpha\) and \(\phi_c)\) are optimized, which is a numerical non-linear multi-objective optimization process. The SA global optimization algorithm can be formulated as solving the following objective function:

Minimize \(\varepsilon(\alpha, \phi_c) = \left| V_m - V_c(\alpha, \phi_c) \right| / V_m \),

(7)

Where \(\varepsilon\) is the absolute value of relative error between the computed velocities \(V_c\) and measured ones \(V_m\). The search space of parameters \(\alpha\) and \(\phi_c\) are \([0, 20]\) and \([0, 1]\) respectively.

(7)Finally, we use the inverted optimal parameters \((\alpha\) and \(\phi_c)\) to calculate the S-wave velocity \(V_{sc}\).

Figure 3 Workflow to predict S-wave velocity based on the effective critical porosity and consolidation coefficient inversion for organic rich rock. \(V_m\) and \(v_c\) denote measured and calculated velocity, \(v_{pc}\) and \(V_{sc}\) are the predicted P- and S-wave velocity.

Model verification

In this paper, a set of organic-rich rock data measured by Vernik and Liu(1997) under high-frequency conditions were used to test the proposed method. These data were measured under ambient conditions and confining pressure of 70MPa, and the samples were saturated rocks. As can be seen from figure 7, the porosity of the shale ranges from 0.0097 to 0.309, and the kerogen content ranges from 0.014 to 0.363.

The elastic constants used in the study are as follows: mineral bulk modulus \(K_m\) is 39.54 GPa, mineral shear modulus \(G_m\) is 25.68 GPa, mineral density \(\rho_m\) is 2.64 g/cm\(^3\), kerogen bulk modulus \(K_k\) is 2.9 GPa, kerogen shear modulus \(G_k\) is 2.7 GPa, kerogen density is 1.3 g/cm\(^3\), fluid bulk modulus \(K_f\) is 2.2 GPa, fluid density \(\rho_f\) is 1 g/cm\(^3\). Figure 4 and Figure 5 show the Cross-plot of the measured velocity versus the predicted velocity using the proposed method, varied critical porosity(VCP) model, varied consolidation coefficient(VCC) model. According to Figure 4, Figure 5, we conclude that the S-wave velocity prediction method proposed in this paper can provide more accurate results than the VCP and VCC method for lab measured data.

Conclusions

In this paper, a new method is proposed to predict S-wave velocity based on the combination of the CPCC rock physics model, the Gassmann equation, the VRH average equation and the SA optimization algorithm. As a result, the lab experiment data applications both indicate that the predicted S-wave velocity by the proposed method agree well with the measured data and are more accurate than that estimated by the VCP and VCC method.
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Figure 4 Cross-plot of the measured velocity versus the predicted velocity using the proposed method, (a)P-wave velocity, (b)S-wave velocity.

Figure 5. Cross-plot of the measured velocity versus the predicted velocity using the VCP method (a and b) and the VCC method (c and d). Data are taken from Vernik and Liu (1997).

References