Introduction

Seismic full waveform inversion (FWI) enables to retrieve subsurface models by using the whole information content of seismic data. Although the high computational cost is still one of the main challenges for the application of FWI, it is nowadays already widely used in various contexts, ranging from the inversion of relatively small-scale near-surface data sets up to large-scale exploration data sets. In the context of exploration seismics, FWI is mainly used to determine velocity models that are needed for imaging methods. A good overview of FWI and recent developments can be found in Virieux et al. (2014).

Besides the high computational costs of FWI, the costs associated with seismic data acquisition can also be substantial. Therefore, it is crucial that during seismic survey design the amount and locations of sources and receivers are selected judiciously. This can be achieved with optimized experimental design (OED). For seismic reflection imaging, the main attributes for the acquisition geometry are fold, offset and spatial sampling to ensure adequate signal levels and a desired spatial resolution. However, for FWI, these criteria are not necessarily the most relevant. Instead, the information contained in the sensitivity kernels needs to be considered (e.g. Djikpesse et al., 2012). Because wave propagation and thus the sensitivities are affected by the velocity structures in the subsurface, it is also important to include prior knowledge about the subsurface when determining an OED.

In this study, we show the limitations of previously suggested measures to quantify the goodness of an acquisition design for FWI. Then, we introduce an alternative measure that overcomes some of these limitations, and we propose an innovative sequential method that allows a computationally affordable determination of optimal source locations. The proposed scheme performance and applicability is demonstrated with a numerical example.

Methodology

To quantify the goodness of a certain acquisition geometry, a suitable objective function or benefit measure needs to be defined. In the context of full waveform inversion, this benefit measure is usually based on the sensitivities contained in the Jacobian matrix $J = \left( \frac{\partial d_i}{\partial m_j} \right)$ with $i = (1,...,N)$ and $j = (1,...,M)$ for $N$ data points and $M$ model parameters. A theoretically ideal benefit measure is the Relative Eigenvalue Range ($RER$) as proposed by Maurer et al. (2009), which is based on the eigenvalue spectrum of the approximate Hessian matrix $J^T J$ and illustrated in Figure 1. Due to the limited illumination of a surface-based acquisition geometry, parts of the eigenvalues will always be zero or close to zero, thereby forming the null space of a certain acquisition geometry (see Figure 1). The goal of OED is to minimize this null space. The benefit of a certain acquisition design will always be calculated in relation to the $RER$ of a reference design, which consists of the maximal possible or desired number of sources and receivers (comprehensive data set). The actual benefit measure is the normalized Relative Eigenvalue Range

$$nRER = \frac{RER}{RER_{\text{Full}}},$$

which is the ratio of the $RER$ of a specific subset of the data and the $RER$ of the full data $RER_{\text{Full}}$. The $nRER$ always lies between 0 and 1, where a value of 0 means that the selected data subset does not contain any information, and a value of 1 means that it has the same information content as the comprehensive data set.

Because firing seismic sources is generally more expensive than deploying receivers, we focus here only on the optimization of source locations for a given number of receivers at fixed positions. To reduce the computational requirements, we employ a sequential OED algorithm (e.g. Guest and Curtis, 2009). To find the optimal source locations, we start by calculating the benefit for all possible sources and chose the one with the highest benefit. In a next step, we chose from the remaining sources the one that adds the most benefit to the already chosen one, and continue until the desired number of sources is reached.
Figure 1 Schematic representation of the eigenvalue spectra of a ‘full’ data set and a subset $S$. The eigenvalue index at a given threshold value defines the respective RER values.

Theoretically, the $nRER$ would be an ideal benefit measure, but the large number of computationally expensive eigenvalue calculations during an OED run precludes this approach from being practical. Therefore, Nuber et al. (2017) instead proposed to minimize the cheaper benefit measure

$$g = \sum_{i=1}^{M} \frac{\text{diag}(J_{\text{Full}}^T J_{\text{Full}})}{\text{diag}(J_{i}^T J_{i}) + \delta}$$

with $\delta$ being a small regularization term. This measure has already been used for OED for acoustic and also elastic shallow-seismic FWI (Nuber et al., 2017). In the following, we will use it for target-oriented OED, where only a small subarea of the model is considered.

Target-oriented OED

We evaluate the performance of the sequential OED algorithm with $g$ as a benefit measure on the subsurface P-wave velocity model shown in Figure 2, which is based on a model created by Gray and Marfurt (1995) as synthetic representation of the geology of northeastern British Columbia. 60 receivers are placed in intervals of 40 m from $x = 70$ m on, and as potential source locations we choose 61 source positions also in intervals of 40 m from $x = 50$ m on. The sources emit a Ricker wavelet pulse with a main frequency of 10 Hz. For the OED calculations, only three discrete frequencies were used, namely 5 Hz, 11 Hz and 18 Hz. In this example, we focus only on a small subarea of the model ranging from $x = 400$ m to $x = 1000$ m and from $z = 300$ m to $z = 600$ m (red rectangle in Figure 2).

We first calculate the optimal source order using the benefit measure $g$. Figure 2 shows the 10 best sources selected with this approach. It is striking that only sources on top of the target area are selected, although those sources provide very similar information and do not allow to resolve the model parameters independently. This result is caused by the fact that $g$ is not sensitive towards linear dependencies in the data and thus ignores data redundancies. It focuses primarily on the magnitude of the sensitivities, i.e. the choice of sources is mainly driven by the amplitudes of $J$. The normalized eigenvalue spectrum of $J^T J$ corresponding to those 10 best sources is shown in Figure 3. It can be seen that, in comparison to the spectra of 10 randomly selected sources (gray curves in Figure 3), the eigenvalues are smaller, meaning that the null space is larger and the design is worse. This becomes even clearer when calculating...
the nRER as a function of the number of sources chosen (Figure 4). The sources selected with g (blue curve) provide a clearly inferior design compared with the random designs (gray curves), demonstrating the necessity of using a different benefit measure for the target-oriented OED.

**An improved benefit measure**

To overcome the previously described problem, we propose the determinant-based measure

\[
g_{\text{det}} = \log \left( \det \left( \frac{J^T J}{\lambda_1^n} \right) \right),
\]

for which the largest eigenvalue \( \lambda_1 \) can be calculated at low cost by using the power method. The exponent \( n \) corresponds to the number of eigenvalues of \( J^T J \). \( g_{\text{det}} \) corresponds to the area under the normalized eigenvalue curve (see Figure 1) and is closely related to the D-criterion known from seismology (Kijko, 1977). Since the determinant is highly sensitive towards linear dependency in the data, this measure is more suitable for target-oriented OED. For that reason Djikpesse et al. (2012) considered a similar measure.
Since the computation of the determinant can still be quite expensive for larger data sets, we determine the optimal sources for each receiver individually and assign weights to every source depending on its relevance for a particular receiver. By summing up those weights for all receivers, we get a ranking of the sources related to their importance for the actual receiver distribution. This receiver-based approach is also computationally attractive because \( \det(J^T J) = \det(JJ^T) \). With the size of \( JJ^T \) depending only on the number of sources \( n_S \), the matrix has a maximal size of \( n_S \times n_S \), regardless of the number of model parameters. This makes the OED calculation with a determinant-based measure computationally affordable. As another advantage, because every receiver is treated separately and independently, this method can easily be parallelized to make it also applicable to large-sized surveys.

We have repeated the experimental design procedure using \( g_{det} \). Additionally, we computed a reference solution, in which we considered the \( RER \) measure. For our small-scale 2D example, this expensive computation was feasible, but our receiver-based approach with \( g_{det} \) reduces the computational time by a factor of 10, even without any parallelization. Because of the computational complexity of the eigenvalue decomposition, this difference will get even more significant for larger amounts of sources and receivers. The 10 best sources obtained with the two approaches are shown in Figure 2. In contrast to the original computations using \( g \), the source locations are spread over the entire surface. For both the \( g_{det} \) and the \( RER \) computations, very similar source patterns were obtained. The similarity of the \( g_{det} \) and \( nRER \) designs can also clearly be seen in the eigenvalue spectra and the \( nRER \) curves in Figures 3 and 4 (black and red lines). Those curves clearly show that \( g_{det} \) provides an optimized design comparable to the ideal but expensive \( nRER \) approach.

Conclusions

We have discussed the limitations of a previously suggested OED objective function and proposed an alternative determinant-based measure \( g_{det} \). This measure is also suitable for target-oriented OED. To reduce the computational requirements, we propose a receiver-based approach, where the optimal sources are calculated separately for each individual receiver instead of considering all receivers simultaneously. With a numerical example, we showed that the determinant-based measure minimizes the null space of the inverse problem. It offers a similar performance as the \( RER \), which is theoretically preferable, but far too expensive for larger scale design problems. Our proposed scheme has the advantage that it is less dependent on the model size. In addition, it can easily be parallelized to be applicable to larger models and larger amounts of sources and receivers as they occur in 3D surveys.

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References


