Angle gathers from time-shift extended least-squares reverse-time migration

Introduction

Quantitative interpretation under complex overburdens like salt remains a challenge, with image amplitudes affected by illumination variations due to the acquisition geometry and overburden velocity complexity. Angle-dependent least-squares reverse-time migration (LSRTM) (e.g., Duveneck et al., 2019) has proven to be a useful tool in such settings, generating reflection angle gathers that are suitable for amplitude variation with angle (AVA) analysis. However, using a shot-based computation of angle gathers during angle-dependent LSRTM (e.g., Duveneck et al., 2021), while being computationally efficient, involves approximations that can lead to inaccuracies in the case of complex wave fields with multi-pathing. In particular, for LSRTM imaging with multiples (e.g., de Vos, 2020), where the source-side wave field per shot is generated by an areal source consisting of the recorded data, the assumptions required for shot-based angle gathers may be violated.

An alternative is to formulate LSRTM with image gathers in terms of subsurface-offset or time-shift extended imaging (e.g. Sava and Fomel, 2003, 2006), followed by a transform to reflection angles. Extended imaging automatically accounts for multi-pathing, without any assumptions on wavefield complexity and uses linear imaging operators with well-defined adjoint (modelling) operators, as required for least-squares migration (LSM). However, while subsurface-offset extended imaging may be an attractive option for wave-equation-based LSM in the case of narrow-azimuth (NAZ) geometries (e.g., Duan et al., 2020), it is computationally expensive for wide-azimuth streamer (WAZ) or ocean-bottom node (OBN) geometries, where well-sampled areal extended images along two horizontal subsurface-offset directions are required for every image point. In such cases, formulating LSRTM based on time-shift extended imaging (Wang and Xu, 2017) may be preferable, since it involves only a single-parameter image extension, even for WAZ or OBN geometries.

In this paper, I present a formulation of iterative time-shift extended LSRTM aimed at producing angle gathers with physically meaningful amplitudes, such that peak amplitudes on a reflector are proportional to the angle-dependent reflection coefficient. I demonstrate the method on a synthetic data example.

Method

The approach presented here involves iterative LSRTM with a time-shift extension, followed by a transform to angle gathers of the final time-shift extended LSRTM result. This requires a number of building blocks, namely an adjoint pair of time-shift extended linearized modelling and imaging operators, an effective time-shift extended-domain preconditioner and a transform from the time-shift domain to the reflection angle domain that properly handles amplitudes.

The adjoint pair of time-shift extended linearized modelling and imaging operators used during time-shift extended LSRTM can be expressed as:

\[
(Lu)(x_r, x_s, t) = \frac{\partial}{\partial t} \int d^3 x \int d\tau \int dt' \left[ G(x, x_s, t' - \tau) u(x, \tau) G(x_r, x, t - (t' + \tau)) \right]
\]

\[
(L^T d)(x, \tau) = \int d^2 x_s \int dt' G(x, x_s, t' - \tau) \int d^2 x_r \int dt \left[ G(x_r, x, t - (t' + \tau)) \right] \frac{\partial}{\partial t} d(x_r, x_s, t),
\]

(1)

where \(d(x_r, x_s, t)\) is the recorded data at receiver position \(x_r\) due to a source at position \(x_s\), \(u(x, \tau)\) is the reflectivity image at location \(x\) for time shift \(\tau\), and \(L\) and \(L^T\) are the modelling operator and its adjoint, respectively. The quantities \(G(y, x, \tau)\) are Green’s functions. Equation (1) shows that time-shift extended linearized modelling involves an integration over time shifts \(\tau\). During this integration, reflector contributions in \(u(x, \tau)\) that correspond to the specular reflection angle for the given source-receiver pair are singled out. Correspondingly, transforming a time-shift dependent reflectivity to the reflection angle domain requires collecting the contributions in the time-shift extended image that belong to the same reflection angle. This means that amplitudes need to be summed along the trajectory defined by the change of reflector position with time shift along the reflector normal, given by

\[
\frac{\partial n}{\partial \tau} = -\frac{v}{\cos \theta'},
\]

(2)
where $v$ is the local velocity, $\theta$ is the reflection angle and $n$ is the coordinate along the local reflector normal $n$. However, directly transforming $u(x, \tau)$, obtained from time-shift extended LSRTM using equations (1), to the angle domain will not yield physically meaningful amplitudes. Linearized modelling from an angle-dependent band-limited reflectivity, defined such that the peak amplitude on a reflector is proportional to the angle-dependent reflection coefficient, involves a factor $\cos^2 \theta/v^2$ (Duveneck et al., 2021). This factor is missing in equation (1) and needs to be accounted for when transforming $u(x, \tau)$ to a reflection-angle dependent reflectivity:

$$u(x, \tau) \rightarrow r_\tau(x, \tau) = v^2(x)u(x, \tau)$$

$$r_\tau(x, \tau) \rightarrow r_\theta(x, \theta) = m(x, \theta) = r_\theta(x, \theta)/\cos^2 \theta. \quad (3)$$

Here, the angle-dependent reflectivity $m(x, \theta)$ is defined such that it can directly be interpreted in terms of angle-dependent reflection coefficients.

To perform the transform $r_\tau(x, \tau) \rightarrow r_\theta(x, \theta)$ from the time-shift to the angle domain based on equation (2), Sava and Fomel (2006) propose a slant stack per time-shift gather, followed by a velocity- and dip-dependent correction. To overcome dip limitations and the need for explicitly specifying a dip field, Vyas et al. (2010) perform the slant stack in the three-dimensional wavenumber domain for a range of $\partial n/\partial \tau$ values, followed by interpolation based on the local velocity. Khalil et al. (2013) apply a velocity-dependent stretch of the time-shift domain. For small shifts, event slopes in the resulting stretched domain then only depend on reflector dip and reflection angle, and a transform to angles can be easily implemented in the wavenumber domain.

However, equation (2) holds only locally. In the presence of velocity variations, a slant stack based on equation (2) will not properly capture all amplitude contributions in $r_\tau(x, \tau)$ belonging to a fixed reflection angle. Correspondingly, a simple stretch of the time-shift axis based on the local velocity does not result in linear events for larger shift values. Instead, I propose applying a velocity-dependent transform of time-shift gathers $r_\tau(z, \tau)$ into gathers $r_\alpha(z, \alpha)$ with an extension coordinate $\alpha$, using the following procedure: For each depth $z_0$ in a time-shift gather, the time-shift/depth trajectory $\tau(z, z_0)$ is computed starting from $\tau = 0$, assuming zero dip and zero reflection angle. Amplitudes along this trajectory are then scaled and mapped to the coordinate $\alpha$ at the current depth $z$ using the velocity at $z_0$:

$$a(z, z_0) = v(z_0)^2(z, z_0) \quad \text{with} \quad \tau(z, z_0) = -\int_{z_0}^z v^{-1}(z') \, dz'. \quad (4)$$

This procedure effectively results in linear events in $r_\alpha(z, \alpha)$ gathers, also for non-zero reflection angle and dip (Figure 1b). For constant velocity, this procedure reduces to a simple stretch, as proposed by Khalil et al. (2013). The resulting image volume $r_\alpha(x, \alpha)$ can then be efficiently transformed to reflection angle (Figure 1c) in the wavenumber $k$ domain:

$$r_\theta(k, \theta) = \int r_\alpha(k, \alpha)e^{-ia|k|/\cos(\theta)} \, da. \quad (5)$$

Figure 1 (a) Example time-shift gathers, (b) corresponding gathers after the mapping procedure of equation (4), (c) corresponding reflection angle gathers.
Figure 2 Sigsbee 2A acoustic wave-equation finite-difference synthetics. (a) Initial time-shift extended migration: zero time-shift section, (b) corresponding selected time-shift gathers ($\tau = -60$ ms to $+60$ ms). (c) Time-shift extended LSRTM after 10 iterations: zero time-shift section, (d) corresponding selected time-shift gathers. (e) Time-shift extended LSRTM after 10 iterations transformed to reflection angles: zero reflection angle section, (f) corresponding selected angle gathers ($\theta = 0^\circ$ to $44^\circ$).

To accelerate convergence of an iterative time-shift extended LSRTM scheme using equations (1), image-domain preconditioning can be applied. A preconditioning operator can be computed from demigration of a suitably defined test reflectivity. In the example shown below, a time-shift dependent preconditioner is computed and applied per time-shift gather. Since equations (1) do not involve a Laplacian operator, images computed by the imaging operator in equations (1) are susceptible to low-wavenumber back-scattering artifacts related to hard velocity contrasts like salt. These artifacts appear around zero time shift in time-shift gathers and are suppressed by the preconditioner.

Example

I demonstrate the procedure of time-shift extended iterative LSRTM followed by a transform to reflection angles on a synthetic data example, namely acoustic finite-difference synthetics computed in the Sigsbee 2A velocity model. The used shot spacing is 100 m with a maximum offset of 10 km in a split-spread geometry. Figures 2a and 2b show the initial time-shift migration. As expected, it is dominated by salt-related low-wavenumber back-scattering artifacts due to the absence of a Laplacian operator in equations (1). The result after 10 iterations of time-shift extended LSRTM with preconditioning is displayed in Figures 2c and d. Transforming the time-shift extended LSRTM image to the angle domain, using the procedure described above, results in the angle-dependent image displayed in Figures 2e and f. The reflection angle range is $\theta = 0^\circ$ to $44^\circ$. Clearly, illumination variation effects on image amplitudes have been corrected by the LSRTM and angle gathers show the expected AVA behaviour. This is confirmed by a comparison with angle gathers obtained with an LSRTM method using a shot-based transform to angle gathers (Duveneck et al., 2021), as shown in Figure 3.
Figure 3 Sigsbee 2A acoustic wave-equation finite-difference synthetics. (a) Selected angle gathers ($\theta = 0^\circ$ to $44^\circ$) from 10 iterations of time-shift extended LSRTM. No muting was applied in angle gathers. (b) Corresponding selected angle gathers from 10 iterations of angle-dependent LSRTM using a shot-based computation of angle gathers (Duveneck et al., 2021). Un-illuminated angles have been muted.

Conclusions

I have presented a method for computing angle gathers using iterative time-shift extended LSRTM. The method aims at producing angle gathers that can be interpreted in terms of angle-dependent reflection coefficients. While being computationally more expensive, the described method potentially handles complex wave fields better than methods using a shot-based transform to reflection angles. The fact that the time-shift domain is a single-parameter image extension can be seen as a strength and a weakness: On the one hand, it makes the method more efficient than approaches using a subsurface-offset image extension in the case of wide-azimuth or ocean-bottom node data. On the other hand, it does not offer any possibility to represent the azimuth-dependence of amplitudes and residual moveout.

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References