Introduction

WEMVA has been widely used for large scale model reconstruction. To make use of full wavefield information and obtain stable inversion results, wave-equation based methods, such as DSO or traveltime inversion (TI), are become popular in recent years. Compared with traditional ray-based methods which is under the assumption of high frequency approximation, wave-equation based tomography methods are intrinsically more robust with high resolution, which can accurately describe wave phenomenon, and thus can solve multipathing problem occurred in ray-based methods especially in dramatically lateral variation areas.

Although effective to handle complex structures, the gradients can be sensitive to the artefacts and uneven amplitude distributions of the images, which would spuriously contaminate inversion results with uneven gradient distributions and decrease the convergent rate. To mitigate the effects of uneven illumination, Lameloise et al. (2015) gave a robust migration velocity analysis by quantitative reverse-time migration (RTM), which can remove the imaging artefacts in offset domain common image gathers (ODCIGs) through ray + Born migration/inversion theory. Hou (2015, 2016) proposed an approximation to the extended Born inverse operator based on stationary-phase theory and applied the theory to DSO inversion. With this extended Born operator, the RTM results can approximately invert the subsurface reflectivity directly, which can eliminate the contamination of the gradients caused by amplitude and artefacts. Other pioneering works also have been done by Chauris et al. (2017) and Qin et al. (2015), to reduce the amplitude affects to the inversion results.

In this paper, we derive a new direct inversion method based on high frequency asymptotic inversion in subsurface offset domain. The new inversion formula is very similar to one in zero offset obtained by Li et al. (2018). Finally, we perform the velocity analysis based on the DSO method with the objective function derived by Shen (2015). We show a different gradient expression based on new inversion method. As expected, the new inversion method introduces angle and amplitude related terms, and thus can eliminate the imaging artefacts caused by uneven illumination. The imaging results and the inverted model can be obtained with artefacts free and amplitude even distributed. We present several 2D numerical examples to illustrate the effectiveness of the method.

Method

In subsurface offset domain, the corresponding Born modeling and the adjoint expression can be written as

$$
\phi[u(x_s,x_r,\omega)] = -\omega^2 \int dx dh \frac{2\delta v(x,h)}{v^4(x)} G(x-h,x_s,\omega) G(x_s,x+h,\omega),
$$

(1)

and

$$
\phi^{-1}[\delta v(x,h)] = -\int dx dh \frac{2\alpha^2}{v^4(x)} G(x-h,x_s,\omega) G'(x_s,x+h,\omega) d(x_s,x_r,\omega),
$$

(2)

where $\phi[u(x_s,x_r,\omega)]$ and $\phi^{-1}[\delta v(x,h)]$ are the forward operator and the inverse respectively, $\delta v(x_s,h)$ is defined as the model perturbation in subsurface offset domain, $G(x_s-h,x_s,\omega)$ and $G(x_s,x_s+h,\omega)$ are the Green’s functions of source and receiver side. Following the similar steps of Chauris et al. (2017), combined with geometrical-optics Green’s function in two dimensions, we can derive the final direct inversion formula

$$
(\Omega^\omega \zeta)^{-1}[\delta v(x,h)] = 4\pi \int dx \frac{A(x) v^4(x)}{A_s v^4(x+h)} \cos \theta_s \frac{1 + v(x+h)v(x)}{v(x-h)} \cos (\alpha_s - \theta_s) [\text{sgn}(\omega)h[\phi[u(x_s,x_r,\omega)]]],
$$

(3)

where $h[\cdot]$ is the Hilbert transform, $\theta_s$ and $v_s$ are the incident angle and velocity at receiver location, sgn is the symbolic function. Rearranging source-receiver wavefields related term, we modify the final imaging condition to

$$
(\Omega^\omega \zeta)^{-1}[a(x,h)] = 4\pi v^4(x) \left( \frac{(i\omega)^2}{v_s^4(x)} U_s^\ast(x-h) W_s(x+h) + \nabla U_s^\ast(x-h) \nabla W_s(x+h) + U_s^\ast(x-h) U_s(x-h) + \gamma^2 \right),
$$

(4)
where $W_r = \partial R(x, x, t)/\partial r_r$ represents derivative of receiver wavefield with respect to depth at receiver locations with $R(x, x, t) = d(x, x, t) \otimes G(x, x, t)$, $U = \text{IFFT}((i\omega)^F \text{FFT}(S(x, x, t)))$ is the weighted source wavefield at source side $S(x, x, t) = f(w) \otimes G(x, x, t)$, $\gamma$ is a stabilization factor, $\nabla$ is the Hamilton operator which represents first-order derivatives with respect to coordinates. For inversion, we choose the objective function proposed by Shen et al. (2015):

$$J = \frac{1}{2} \| w (\frac{\partial I}{\partial v} - \Delta I) \|,$$

where $\Delta I$ is the residual images which can be approximately calculated by $\Delta I = \partial I / \partial h \Delta h$, $w$ is the weighting operator. We can use the adjoint state method (Plessix, 2006) to derive the gradient which is not shown here. Associated to the modified imaging condition (equation 4), the gradient formula is obvious distinct from the traditional one (Shen et al., 2015) by considering angle and amplitude information contained in the wavefields. As the linear relationship between the model perturbation $\partial v$ and the image residuals $\Delta I = \partial I / \partial h \Delta h$, the model can be updated directly through conjugate gradient method. Combined with the modified imaging (equation 4) and the corresponding inversion operators, we expect a robust inversion process.

**Examples**

We start with a simple homogeneous model with a single reflector embedded at the depth of 1.0 km with true velocity 2000 m/s. The space interval is 10 m either in vertical or horizontal direction. We trigger 145 shots with 0.02 km interval per shot from 0.05 km to 2.96 km on the surface using Ricker wavelet with the central frequency of 15 Hz. Total 291 receivers are evenly distributed with 10 m interval on the surface. Firstly, we migrate the simulated data in subsurface offset domain (101 offsets with offset interval 10 m) with true velocity by using the traditional cross correlating and the new

**Figure 1** ODCIGs obtained by traditional cross correlating imaging condition with correct velocity model a–c and lower velocity model g–i and our inversion method with correct velocity model d–f and lower velocity model j–l at the location of 0.5 km (a, d, g, j), 1.5 km (b, e, h, k), 2 km (c, f, i, l).

inversion method respectively. The ODCIGs are extracted at the location of 0.5 km, 1.0 km and 1.5 km, respectively, shown in Figure 1 a–c (traditional) and d–f (new). Then, to verify the effectiveness of the algorithm on the gradient, we migrate the simulated data by using a homogeneous model with velocity 1900 m/s. The corresponding ODCIGs extracted at the same locations are shown in Figure 1 g–i and j–l. By contrast, regardless of the velocity model (correct or not), the imaging results of the new method are less contaminated by spurious artefacts related to uneven illumination. For inversion,
we apply traditional and new method respectively to the ODCIGs with corresponding gradients at first iteration shown in Figure 2a (traditional) and Figure 2b (new). After 30 iterations, the inverted model of the new method performs more evenly distributed and artefacts free results (Figure 2d) compared with the traditional one (Figure 2c).

![Figure 2](image)

**Figure 2** Gradients of 1st iteration in the traditional (a) and the new inversion (b) method respectively with corresponding final inverted models for the traditional (c) and for the new method (d).

We apply land field data to further verify the flexibility and robustness of the proposed method. The background velocity is shown in Figure 13. There are 500 shots unevenly distributed on the surface with 204 receivers per shot in an interval of 50 m. The recording length is 4 s with 2 ms sampling interval. The minimum phase wavelet (Figure 3a) is estimated by the method proposed by Pratt (1999). We migrate the data with an initial velocity model shown in Figure 3b and extract the ODCIGs at the locations of 12.0–20.0 km with offset interval 15 m. Compared to the ODCIGs obtained with the traditional method (Figure 5a), the image gathers with the new method (Figure 5b) appear more balanced amplitude and less artefacts. We implement the iterative algorithm based on the conjugate gradient method within 12.0–20.0 km of the model to inversion. After 30 iterations, the corresponding inverted models are shown in Figure 4. In contrast, with the same iterations, the inverted model (Figure 4b) based on the new method shows more details than the traditional one, which means that the new method can gain a faster convergence as the curves shown in Figure 6. The corresponding image gathers corresponding to the inverted models in Figure 4 are displayed in Figure 5c–d. In contrast, the migrated image based on new method contains more focused events and reveals more structural details.

![Figure 3](image)

**Figure 3** (a) The minimum phase wavelet and (b) initial velocity model used for inversion.

![Figure 4](image)

**Figure 4** Inverted models with conventional (a) and new (b) method.
Conclusions

In this paper, we derive a direct inversion method to mitigate the noises presented in the stacked images and inversion results. The theoretical basis is a high frequency asymptotic of Hessian matrix, which introduces angle and amplitude related terms in subsurface offset domain. Synthetic and field data illustrate the effectiveness of the method.

Acknowledgements

This research is supported by the National Natural Science Foundation of China (NSFC) under contract number 41874144.

Reference