On the stable implementation of the obliquity factor operators for 3D marine processing

Introduction

Many seismic processing algorithms operate on multidimensional, multicomponent data to account for the 3D nature of the seismic experiment and to allow for broadband processing. For instance, in towed-marine streamer processing, by combining pressure and accelerometer measurements from multiple streamers, the pressure wavefield can be dehosted and interpolated (Özbek et al., 2010). Similarly, in ocean-bottom processing, combining pressure and vertical acceleration measurements in common receiver gathers allows removal of the receiver ghost and increases the temporal bandwidth (Kristiansen et al., 2015).

A common step in processing multidimensional multicomponent data is removing the obliquity factor (associated with the vertical wavenumber) from the accelerometer data. Although the obliquity factor has a simple mathematical expression in the continuous frequency-wavenumber domain, due to the notch at water velocity, its removal from discrete-space, finite-aperture acceleration data requires special care. There exist several mitigation strategies to avoid the notch of the inverse obliquity factor such as: adding a white noise gain to the denominator of the operator to dampen the infinite resonance at water velocity, or truncating the unstable operator at a velocity larger than the water velocity. Each of these approaches have some shortcomings. For instance, adding the white noise term changes the operator response at all frequencies and wavenumbers, and truncation leads to the Gibbs phenomenon. Perhaps, more importantly, the modified operator typically has an infinite impulse response, and its application to discrete data in the transform domain causes ringing and wrap around artefacts. Although zero padding helps reduce the wrap around artefacts, the optimum padding length is not always known beforehand.

Applying a continuous operator to a discrete-space signal is a common problem in signal processing. One approach is to interpolate the discrete-domain signal to continuous-domain by using the sampling theorem, and applying the continuous operator. The second approach is to approximate the continuous domain operator with a discrete operator whose discrete Fourier transform approximates the continuous domain operator within Nyquist bandwidth. The second approach is preferable as the discrete filter can be designed once and applied to any data. In this abstract, we show that the impulse responses of the continuous domain operators such as obliquity factor, inverse obliquity factor, and fan filter have simple analytical expressions. We show that, although the inverse obliquity factor is unstable at water velocity, the corresponding impulse response is well-behaved and decays slowly in space. Then, by using the finite impulse response (FIR) filter design by the windowing approach, we approximate the continuous domain response in the discrete space domain. The size of the FIR filter controls the accuracy approximation and the amount of required padding. We illustrate on a synthetic data example that this way of applying the obliquity/inverse obliquity operators achieves artefact-free and an accurate approximation to the corresponding continuous domain operators.

Theory

Consider a two-dimensional operator with Fourier transform \( H(k_x,k_y) \), where \( k_x \) and \( k_y \) are the horizontal wavenumbers. The impulse response of the operator in the space domain is computed by the inverse Fourier transform:

\[
h(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} H(k_x,k_y) e^{2\pi i (k_x x + k_y y)} \, dk_x \, dk_y .
\]  

For a circularly symmetric transform, it is more convenient to write (1) in polar coordinates:

\[
h(r,\theta) = \int_{0}^{2\pi} \int_{0}^{\infty} H(k_x,\theta) e^{2\pi i (k_x r \cos \theta + k_y r \sin \theta)} \, dk_x \, dk_y
\]

where \((x,y) = (r \cos \theta, r \sin \theta)\) and \((k_x,k_y) = (k_r \cos \phi, k_r \sin \phi)\). When \( H \) is circularly symmetric, we can write \( H(k_x,\theta) = H(k_r), \) and the integral in (2) becomes independent of \( \phi \):

---

82nd EAGE Annual Conference & Exhibition
h(r) \triangleq h(r, \theta) = \int_0^\infty H(k_r) k_r \left[ \int_0^{2\pi} e^{2\pi i k_r \cos \phi} \, d\phi \right] \, dk_r.

The integral over \phi gives the zero-order Bessel function of the first kind, J_0, (3) becomes

h(r) = \int_0^\infty H(k_r) k_r J_0(2\pi k_r r) \, dk_r.

This is known as the Hankel transform. Table 1 shows the analytical expressions for the Hankel transforms of common circular symmetric operators encountered in seismic processing. These include velocity filter (fan filter), the obliquity factor, and the inverse obliquity factor.

<table>
<thead>
<tr>
<th>Fourier transform</th>
<th>Impulse response</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fan filter</td>
<td>\text{circ}(k_c/f)</td>
</tr>
<tr>
<td>Obliquity factor</td>
<td>\left(1-k_c^2 c^2 / f^2\right)^{-1/2} \text{circ}(k_c/f)</td>
</tr>
<tr>
<td>Inverse obliquity factor</td>
<td>\left(1-k_c^2 c^2 / f^2\right)^{-1/2} \text{circ}(k_c/f)</td>
</tr>
</tbody>
</table>

Table 1 The 2D Fourier representations of some circular symmetric responses (middle) and the corresponding impulse responses (right). Here, \( f \) is frequency and \( c \) is velocity, \( J_1 \) is the first-order Bessel function of the first kind and \text{circ}(\cdot) is the generalization of the boxcar function to higher dimensions: \text{circ}(k_c/f) is 1 if \( |k_c/c|<1 \), and 0 otherwise.

The impulse responses given in Table 1 are exact analytical expressions. We note that the impulse responses of the velocity filter and inverse obliquity factor decay by \( 1/r \), whereas the impulse response of the obliquity factor decays by \( 1/r^3 \). The FIR filter design technique approximates the infinite impulse response filter by applying a window to the ideal response:

\[ h_w(r) \triangleq h(r) w(r), \]  

where the window \( w \) could be chosen among Hamming, Hann, or Kaiser windows or one of commonly used windows in FIR filter design. Figure 1 illustrates the FIR filter approximations for the fan filter, obliquity factor, and inverse obliquity factor at 25 Hz and 1500 m/s. Plots 1(a), 1(b), and 1(c) show the windowed impulse responses; plots 1(d), 1(e), and 1(f) show the corresponding 2D Fourier transforms; and 1(g), 1(h), and 1(i) compare the \( k_r = 0 \) slice of the ideal responses and the corresponding FIR filter approximations. The top row confirms that the impulse response of the obliquity factor decays faster than the other two impulse responses, and the bottom row confirms that the FIR approximations give an accurate approximation to the continuous domain responses over a large wavenumber range. This is particularly important for the inverse obliquity factor which has a singularity at \( k_r = f/c \), which corresponds to the P-wave travelling horizontally.

**Synthetic ocean-bottom node data example**

In this section, we study the effect of the derived FIR filter approximating the inverse obliquity factor on a synthetic data set modelled for an ocean-bottom node (OBN) configuration. We generated the pressure and vertical velocity wavefields using a finite-difference modelling tool. The model is representative of a Gulf of Mexico complex geological setting with a water depth of 1430 m. For this example, we use a single common receiver gather computed for a regularly sampled source grid of 25 m x 25 m and a maximum aperture of \( \pm 2500 \) m. The data bandwidth is limited to 30 Hz.

We separate the upgoing and downgoing wavefields using PZ summation and then scale the upgoing wavefield by \(-1/2ik_v\) (where \( k_v \), the vertical wavenumber, equals the obliquity factor multiplied by water velocity/frequency) as a preliminary step to up/down deconvolution. Deconvolving the upgoing wavefield from the downgoing wavefield results in the earth’s reflectivity (Amundsen et al., 2001). By dividing by \(-2ik_v\), we assign a monopole source response to the reflectivity. Figures 2(a), 2(b), and 2(c)
show the upgoing wavefield divided by \(-2ik_z\) in the time-space \((t,x,y)\) domain and the frequency-space \((f,x,y)\) domain. The input pressure and vertical velocity are padded in the \((t,x,y)\) domain. The PZ summation and the scaling by \(-1/2ik_z\) are applied in the \((f,k_x,k_y)\) domain. For comparison, Figures 2(d), 2(e), and 2(f) show the upgoing wavefield multiplied by the FIR filter approximating \(1/2ik_z\). Note that the discussed artifacts resulting from the infinite length of the impulse response occur not only in space, but also in time. As we are applying the inverse obliquity factor as a 3D operator, windowing it in time suppresses the high-energy linear events appearing at early times denoted by the black arrow on the difference plots in Figure 2(g). The difference time slice (Figure 2(h)) highlights the wrap-around artefacts removed by the FIR filter.

Conclusions

We derived analytical expressions for the impulse response of the obliquity factor, its inverse and a fan filter by taking advantage of their circular symmetry. These analytical representations are used to compute discrete window-based FIR filters to approximate their continuous domain transforms. The approximation accuracy depends on the window length. Windowing helps reduce filtering artefacts by ensuring that the filter has a finite response in the space domain. We demonstrated the effectiveness of this method by applying the FIR filter approximating the inverse obliquity factor to multidimensional multicomponent synthetic common-receiver gather simulating an OBN experiment. We combine the modelled pressure and vertical velocity to decompose the pressure into upgoing and downgoing
wavefields. The upgoing wavefield is subsequently scaled by the inverse obliquity factor. The wavefield scaled by the FIR filter approximating the inverse obliquity factor contains significantly fewer artefacts in the time-space and frequency-space domains than the wavefield scaled by the inverse obliquity factor.

**Figure 2** Upgoing wavefield scaled by obliquity factor: (a) 2D slice in time-space domain, (b) time slice in space domain, and (c) 2D slice in frequency-space domain. Upgoing wavefield scaled by FIR filter: (d) 2D slice in time-space domain, (e) time slice in space domain (orange arrow) and (f) 2D slice in frequency-space domain. Difference plots: (g) 2D slice in time-space domain, (h) time slice in space domain and (i) 2D slice in frequency-space domain.

Acknowledgements

We thank WesternGeco Multiclient for permission to use the data.

References

