Introduction

The objective of reservoir characterization is to understand the reservoir rocks and fluids through accurate measurements to guide and support asset team decisions. In that sense, seismic inversion plays a dominant role in providing the structural framework and an estimation of rock properties, with accuracy depending on the sensitivity of surface seismic data to petrophysical parameters (Nascimento et al., 2014).

The advent of the Controlled-Source EM (CSEM) methodology, producing an estimate of the resistivity, brought the multiphysics data into the ‘seismic-dominated’ characterization workflow and significantly enhanced the accuracy of the estimated rock and fluid properties. The effectiveness of a deterministic inversion scheme leveraging the availability of a multiphysics dataset has been demonstrated by case studies from different locations on the globe: Miotti et al. (2013) from the Barents Sea, Medina et al. (2015) from Borneo, Rovetta et al. (2017) from Saudi Arabia, and Zerilli et al. (2017) from offshore Brazil. In addition, statistical rock physics modelling (Avseth et al., 2001) was introduced to the interpretation community as a tool to address the uncertainty associated with the rock-physics analysis. Bachrach (2006) described how the Bayesian estimation framework can be exploited to estimate porosity and saturation from attributes and well logs through rock physics modeling, and Hokstad and Tanavasuu-Milkeviciene (2017) demonstrated the general validity of the multi-geophysical Bayesian framework by its application to a geothermal exploration scenario.

This work constitutes a first step towards generalizing a multi-geophysical framework that, independently from the choice of the inversion strategy (i.e., deterministic or statistical), can exploit all the available geophysical measurements and properties for an improved understanding of reservoir uncertainties.

The two workflows are evaluated on an offshore oil field where a multi-physics dataset (3D seismic and 3D CSEM data) and well logs are available.

Deterministic and statistical inversion workflow

We briefly introduce the deterministic and the statistical inversion workflows. Without loss of generality, we will assume the porosity, $\phi$, and the water saturation, $S_w$, as model parameters, while the compressional velocity, $V_p$, and the electrical resistivity, $R$, will constitute our observations. For the deterministic implementation of the workflow, we follow Tarantola’s approach on inverse problems (Tarantola, 2005). We start from the non-linear relationship between the model parameters and the observations:

$$d = g(m) \iff \left[ V_p \right] = g(\phi, S_w).$$

The function $g$ is constituted by a set of rock models: Gassmann’s equation (Gassmann, 1951) is used to obtain the elastic moduli from the porosity, $\phi$, and the water saturation, $S_w$, material averaging provides the density and, finally, the Simandoux relationship (Simandoux, 1963) is used to compute the electrical resistivity. With reference to Figure 1a that depicts the high-level diagram of the deterministic inversion workflow, the first logical step is the calibration of the composite rock model, $g$, using the available well-log data to describe the reservoir formation through an optimization procedure. Once the rock model is calibrated, it is used in the forward modeling operator during the deterministic inversion step where we assume Gaussian probability distribution for both model parameters and data. According to the Bayesian theory, the state of information on the model parameters is described by the prior model, $m_0$, and by the model covariance, $C_m$ matrix, while the uncertainty of the observation is captured by the data covariance matrix, $C_d$. The solution of the inverse problem is obtained through an iterative procedure that linearizes the forward model around the current model, $m_k$, and obtains a new model, $m_{k+1}$. The Jacobian matrix, $G_k$, that contains the derivatives of the composite rock model with respect to the values of the model parameters at the $k$-th iteration is evaluated numerically and the solution is updated according to:

$$m_{k+1} = m_k - \left[ G_k^T C_d^{-1} G_k + C_m^{-1} \right]^{-1} G_k^T C_d^{-1} \left( g(m_k) - d \right) - C_m^{-1}(m_k - m_0).$$
The iterative algorithm stops when the update of model parameters produced at the \(k\)-th iteration is less than the convergence threshold, \(\varepsilon\):

\[
\|\mathbf{m}_{k+1} - \mathbf{m}_k\| < \varepsilon.
\]

The uncertainty of the solution is captured by the posterior model covariance matrix expressed as:

\[
C_{m,post} = [G_k^T C_d^{-1} G_k + C_m^{-1}]^{-1}.
\]

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**Figure 1** (a) Deterministic inversion workflow. (b) Statistical inversion workflow.

Similar to the deterministic workflow, the first stage of the statistical workflow aims to estimate the elastic and electromagnetic responses of the effective media characterized as porosity and fluid saturation (Figure 1b). The rock model calibration is followed by the estimation of the joint a-priori Probability Density Function (PDF) of the petrophysical properties that constitute the model parameters to be inferred by the inverse process. Subsequently, the samples of the random variables (i.e., porosity and water saturation) can be drawn from the PDF previously estimated using a Markov Chain Monte Carlo (MCMC) technique based on the Metropolis-Hastings algorithm (Hastings, 1970). The results of the MCMC simulation serve as input to the calibrated rock model and are used to compute the random ‘observations’ of compressional velocity and resistivity, i.e., constructing the likelihood distribution. After completing the simulation, it is possible to compute the four-dimensional posterior PDF that relates the model to the data. Following Bayes’ theorem, the posterior probability of the model parameters given the data is expressed as:

\[
P(\phi, S_w | V_p, R) = P(V_p, R | \phi, S_w) \cdot P(\phi, S_w) \cdot P(V_p, R),
\]

At each point on the three-dimensional grids where the compressional velocity and resistivity are defined, porosity’s and water saturation’s posterior PDF is calculated. Once the Maximum A Posteriori (MAP) is derived, the P90, P50, and P10 quantiles, which are traditionally exploited in the petroleum industry to forecast reserve values at three confidence levels, can also be computed and used to quantify the uncertainty associated to the Bayesian estimation process. It is important to note that same quantiles can derived for hydrocarbon saturation directly from water saturation results.

**Field case**

The two workflows were applied on the Johan Castberg oilfield data (Barents Sea, Offshore Norway). The composite rock model linking porosity and water saturation to compressional velocity and electrical resistivity was calibrated using log data from three different wells. The reliability and the accuracy of both inversion approaches were validated using well data and the porosity and water saturation estimates obtained from inverting P-sonic and resistivity logs are in excellent agreement with the benchmark petrophysical logs. Subsequently, the same procedures were tested on the 3D multi-physics dataset consisting of a compressional velocity cube derived from a full waveform inversion and a vertical resistivity cube.
obtained from the inversion of CSEM soundings with the objective of estimating the spatial distribution of porosity and water saturation. The observations and the model parameters along a polyline connecting the three wells are depicted in Figure 3. As expected, the CSEM resistivity (Figure 2a) correlates very well with the hydrocarbon saturation (Figure 2c). The predicted porosity (Figure 2d) is naturally correlated with the compressional velocity (Figure 2b) but shows also some of the traits that are interpretable on the resistivity section, thus revealing the impact of the ‘joint’ approach. Exploiting the correlation between porosity and resistivity is extremely valuable as it enables the decoupling of the elastic effects from the fluid effects when interpreting the P-velocity drops near the top of the reservoir (Figure 2b).

Figure 2 Observations and estimated model parameters along a polyline connecting the three wells. (a) Vertical resistivity from CSEM inversion. (b) Compressional velocity from FWI. Hydrocarbon saturation (c) and porosity (d) are the MAP estimates obtained with the statistical inversion workflow.

Figure 3 Maps of the hydrocarbon saturation at reservoir top displayed with the hydrocarbon flag log for the three wells available. (a) MAP estimate of the hydrocarbon saturation. (b) P90 quantile of hydrocarbon saturation.
Conclusions

Deterministic and statistical joint seismic and CSEM workflows were applied to 3D cubes of resistivity and velocity to simultaneously recover porosity and water saturation distribution in the prospect. Both workflows start from the definition and the validation of the rock model using well log data. Then, the deterministic approach exploits the rock model as forward operator directly in the iterative inversion process while in the statistical approach is used to produce the observations from the sampled probability distribution. The statistical workflow is, in general, more complex, but it is richer for what concerns the descriptive capabilities of the uncertainty of the inversion results; with respect to the standard deviation, the quantiles are more effective in supporting the assessment of the possible scenarios and, therefore, the decision-making process. Finally, if the quantities involved in the inverse process are expected to have a multimodal distribution, the statistical workflow should be preferred as it does not rely on any assumption of their probability distribution.

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References