The Joint Imaging of Primary and Internal Multiples

Introduction

Multiples are also real reflections from the underground interface, and if internal multiples can be effectively used, it can assist in illuminating shadowed areas that cannot be reached by primary. However, the researches on internal multiples imaging are faced with the problems of high computational cost and difficult to remove the artifact. Therefore, how to carry out the research on the joint imaging method of primary and internal multiples without separating the internal multiples are of great significance.

A prestack depth migration method using model-based internal multiple estimation was proposed (Young, 2001). Full-wave field migration can make the multiples image simultaneously with primary imaging (Berhout, 2011). Multiples imaging based on the wavefield separation by SRME and focusing transform was carried out further (Li, 2015), and the multiples reverse time migration method was justified, which was based on primary and multiples separation of all orders. The RTM method capable of processing internal multiples was proposed by extrapolating the source and receiver wavefields into upward and downward waves in order to cross-correlation imaging (Liu et al., 2015). An efficient method based on Marchenko equation was proposed to image the whole wavefields including internal multiples (Singh et al., 2014). In this paper, it is firstly verified that reverse time migration can image not only primary, but also internal multiples, and further we analyse the feasibility of the joint imaging of primary and internal multiples. Then, the problems of opposite polarity between primary and internal multiple imaging are studied, and the consistency corrections of imaging polarity are carried out, so that the energy at the imaging interface are effectively compensated.

Method

1 Image conditions for migrated primary and internal multiples

2D reverse time migration is based on the following acoustic equation:

\[
\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 p}{\partial t^2}
\]  

(1)

The corresponding assumption conditions and the up-boundary conditions of inverse time extrapolation can be expressed as

\[
p(x, z, t)|_{t=\tau_{\text{max}}} = 0
\]  

(2)

\[
p(x, z, t)|_{t=0} = \phi(x, t)
\]  

(3)

Where \(v\) is the velocity of longitudinal wave, \(\phi(x, t)\) is shot recording. Reverse time migration of 2D acoustic wave equation can be realized by solving the above equation.

In the reverse time migration of internal multiples, internal multiple imaging can be achieved by using crosscorrelation imaging conditions. In the same way, primary imaging can also be realized. Therefore, this paper studies the joint imaging method based on crosscorrelation imaging conditions. Considering the particularity of internal multiples, constructing the imaging conditions of the up and down wavefields decomposition are an effective means to eliminate the offset noise in the imaging process. Therefore, imaging conditions can be written as

\[
I(r) = \sum_{t=0}^{T} S(r, t) R(r, T-t)
\]

\[
= \sum_{t=0}^{T} [S_U(r, t) + S_D(r, t)] R_U(r, T-t) + R_D(r, T-t)
\]

\[
= I_{UU}(r) + I_{DD}(r) + I_{UD}(r) + I_{DU}(r)
\]  

(4)
Where subscripts U and D indicate the direction of upward and downward propagation, respectively.

Pairs of upward source and upward receiver wavefields provide effective imaging at the interface, while Noises are created by unpaired crosscorrelation of wavefields. Therefore, the imaging conditions based on separation of wavefields are expressed as:

$$ I = I_{UL} + I_{DD} $$

(5)

In order to remove the crosstalk noises in the imaging, the wavefields must be separated. F-K filtering and Poynting vector methods are generally used to separate up and down wavefields.

2 Feasibility Analysis of Imaging

The feasibility of internal multiples reverse time migration and the joint reverse time migration of primary and internal multiples are verified in this section. In order to enhance the energy of internal multiples, the velocity model as shown in Figure 1(a) is designed.

![Figure 1](image1.jpg)

**Figure 1** Trial calculation of flat layer model. (a) Velocity field; (b) single shot recording; (c) internal multiples.

![Figure 2](image2.jpg)

**Figure 2** RTM imaging analysis of internal multiples. (a) $T = 0.36s$ imaging process; (b) $T = 0.48s$ imaging process.

![Figure 3](image3.jpg)

**Figure 3** RTM imaging analysis of primary and internal multiples. (a) $T = 0.36s$ imaging process; (b) $T = 0.48s$ imaging process.

The schematic diagram of the internal multiple imaging process is showed at $t=0.36s$ and $t=0.48s$ in Figure 2, respectively. In Figure 2(a), from left to right, there are snapshots of forward and backward extrapolation, and in first interface, the imaging results of crosscorrelation between the two wavefields at $t=0.36s$. In Figure 2(b), the corresponding imaging process are exhibited from left to right at $t=0.48s$. It is indicated that internal multiples can be used to image the second interface. Here, only two special moments are listed to verify the conclusion that RTM can image internal multiple. It is not difficult to
prove that the conclusion is correct according to the analysis of imaging process at all moments. Similarly, the theoretical based on the joint reverse time migration of primary and internal multiples are verified effectively.

Similar to primary, internal multiples can also be effectively imaged, according to the Figure 6, and more underground structure information can be obtained on both sides. However, in the imaging results, the artifact of crosstalk are clearly visible, which are very bad for subsequent interpretation.

![Figure 4](image1.png)

**Figure 4** The imaging results. (a) The results of internal multiples RTM imaging; (b) the results of primary and internal multiples RTM imaging.

3 Conformance correction for imaging polarity

The imaging energy values $m_{ig}$ at each imaging point $x$ at each time are recorded, the imaging polarity of primary is $\text{sign}(1)$ on record. Other imaging polarities are recorded for $\text{sign}(i)$. If the polarity is opposite, the imaging polarity at that moment is corrected.

$$M_{ig} = \begin{cases} m_{ig} & \text{sign}(i) = \text{sign}(1) \\ (-1)^{*}m_{ig} & \text{sign}(i) \neq \text{sign}(1) \end{cases}$$ \hfill (6)

![Figure 7](image2.png)

**Figure 7** Comparison of results before and after polarity correction of interface imaging of a single shot at a certain time. (a) Primary imaging ($t=235\text{ms}$); (b) internal multiples before polarity correction ($t=390\text{ms}$); (c) internal multiples imaging after polarity correction ($t=390\text{ms}$).

By comparing Figure 6(a) and (b), it can be seen that the imaging polarity of internal multiples are opposite with the joint imaging results. There are the local magnification of the imaging results of single shot recording at the position of the first reflecting interface in Figure 7. In Figure 7(a), the imaging contribution from primary reflection at a certain time is displayed. It is showed the imaging contribution from internal multiples at a certain time before polarity correction in Figure 7(b). It is clearly showed that the imaging polarity is opposite, in the same reflecting interface. In Figure 7(c), there is the imaging contribution results after polarity correction. Obviously, after polarity correction, they describe the same reflecting interface with the same polarity.

In order to further verify the feasibility and effectiveness of this method, we further compared the single channel waveform (the 201th channel) of the joint imaging results of multi-shots before and after polarity correction in Figure 9.

![Figure 9](image3.png)

As a comparison between Figure 9(a) and (b), the energy of the imaging interfaces are significantly enhanced after the polarity correction of the joint imaging, which mainly comes from the imaging
The contribution of internal multiples and are labelled in the red box. The polarity corrected imaging results obtained by using the traditional cross-correlation imaging conditions is presented in Figure 9(c). Compared with Figure 9(b) and (c), generated by the wavefield separation imaging conditions, the energy of the illusion are significantly weaker than the traditional cross-correlation imaging conditions in the blue box. Therefore, the polarity correction method used in this paper can well solve the problem of inconsistency between primary and internal multiple imaging correction.

**Figure 9** Waveform comparison of the 201th imaging channel of multi-shots joint imaging. (a) Before polarity correction of joint imaging; (b) after polarity correction of joint imaging; (c) conventional imaging conditions after polarity correction for joint imaging.

**Conclusions**

In this paper, the method of joint imaging of primary and internal multiples based on reverse time migration is discussed, and the feasibility of imaging is analysed, and the problems of polarity in imaging are well solved. At the same time, the SNR of the imaging profile can be improved by the imaging condition of wavefield separation crosscorrelation. However, there is still obvious illusion of crosstalk in the imaging results, which requires further research on the mechanism of illusion generation and suppression methods.

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**References**