On the inverse propagation of surface receiver wavefields in 1.5-dimensional joint migration inversion

Introduction

By exploiting multiples for inversion, joint migration inversion (JMI) aims at simultaneously dealing with velocity model building and seismic imaging (Berkhout, 2014b). The key of JMI is in its unique modelling engine, which is named full wavefield modelling (FWMod) (Berkhout, 2014a). FWMod is based upon one-way propagation operators (Sun et al., 2018a), and it uses imaging parameters, including a velocity model, a reflectivity model and a transmission model, as the input. JMI aims at minimizing the mismatch between simulated data and measurement, and its inversion scheme is based upon the gradients of all the models engaged in FWMod (Sun et al., 2019). Due to practical challenges, the state-of-the-art JMI adopts over-simplified operators in its FWMod engine, and hence the amplitude-versus-offset (AVO) effect in data cannot be adequately addressed (Sun et al., 2020).

In this paper, we consider the application of JMI in 1.5-dimensional (1.5D) media. Furthermore, we treat a velocity model and a density model as unknowns in 1.5D JMI for inversion (Santosa and Symes, 1985). In this particular situation, as one-way propagation, transmission and reflection operators can be analytically defined using medium velocity and density information, the AVO challenge can thus be resolved in 1.5D JMI. By deriving the theoretical framework for 1.5D FWMod, we propose a new concept of ‘inverse propagation’ in 1.5D JMI. We formulate the theory behind this new concept in detail, and further use a synthetic model to demonstrate its correctness.

1.5D FWMod and the inverse propagation in 1.5D JMI

We consider layered media for 1.5D FWMod and 1.5D JMI, and the wavefield-propagation model is shown in Figure 1. Please note that all the wavefield-related symbols on Figure 1 are actually subject to a certain combination of $k_x$ and $\omega$, although we leave them out to save space. For example, $q^+(z_m)$ there actually represents $q^+(z_m; k_x, \omega)$.

Our theories for both 1.5D FWMod and 1.5D JMI are derived in the temporal frequency – spatial wavenumber (FK) domain. We consider a pressure signal corresponding to a combination of a $\omega$-frequency and a medium with a velocity $c$, and we do not consider anisotropy, so we can just set $\omega$ spatial wavenumbers in the $y$ direction,

$$\omega = 2\pi f, \quad k = \frac{\omega}{c}, \quad k_x^2 + k_z^2 = k^2. \tag{1}$$

Using symbols shown in Figure 1, the theory for 1.5D FWMod can be written as follows:

$$q^+_n(z_m) = s^+(z_m) + t^+(z_m) \cdot p^+_n(z_m) + r^+(z_m) \cdot p^-_{n-1}(z_m), \tag{2}$$

$$q^+_n(z_m) = s^-(z_m) + t^-(z_m) \cdot p^+_n(z_m) + r^-(z_m) \cdot p^-_{n}(z_m), \tag{3}$$

$$p^+_n(z_m) = w(z_m, z_{m-1}) \cdot q^+_n(z_{m-1}), \tag{4}$$

$$p^-_n(z_m) = w(z_m, z_{m+1}) \cdot q^-_n(z_{m+1}), \tag{5}$$

$$r^-(z_m) = -r^+(z_m), \tag{6}$$

$$t^+(z_m) = 1 + r^-(z_m), \tag{7}$$

$$t^-(z_m) = 1 + r^+(z_m), \tag{8}$$

$$w(z_m, z_{m+1}) = w(z_{m+1}, z_m), \tag{9}$$

where $n$ is the multiple order (or the iteration order), + or – represents the down-going or the up-going direction, $z_m$ is the depth of the $m$th interface in the $z$ direction, $p$ is an incoming wavefield, $q$ is an outgoing wavefield, $t$ or $r$ is a transmission or reflection coefficient, and $w$ is a one-way propagator. In layered media, $w(z_m, z_{m+1})$ and $r^-(z_m)$ have been discussed before (Berkhout, 1987; Sun et al., 2018a), and they can be written as:
\[ w(z_{m+1}, z_m) = \begin{cases} \exp(-j \cdot k_{z,m} \cdot |z_{m+1} - z_m|) & \text{if } k^2 \geq k_x^2 \\ \exp(-|k_{z,m} \cdot |z_{m+1} - z_m|) & \text{otherwise} \end{cases}, \]

\[ r^- (z) = \begin{cases} \rho_m k_{z,m-1}^{-1} \rho_{m-1} k_{z,m-1} \rho_{m-1} k_{z,m} & \text{if } k^2 \geq k_x^2 \\ \rho_{m-1} k_{z,m-1}^{-1} - \rho_{m-1} k_{z,m} & \text{if } k^2 < k_x^2 \end{cases}, \]

\[ k_{z,m} = \sqrt{k_m^2 - k_x^2}, \]

where \( k_m = \frac{\omega}{c_m}, c_m \) or \( \rho_m \) is the velocity or density of the layer between \( z = z_m \) and \( z = z_{m+1} \).

**Figure 1** The fundamental wavefield-propagation model for 1.5D FWMod and 1.5D JMI.

In seismic data acquisition, sources are normally located at the surface. In addition, people commonly estimate a source wavefield that only contains downward propagating components, i.e. \( s^-(z_m) = 0 \) for all \( z_m \) and \( s^+(z_m) = 0 \) for \( z_m \neq z_0 \). Furthermore, we usually can only measure wavefields at surface. In JMI, one crucial component is to reconstruct wavefields at a certain depth level from a surface receiver wavefield (Sun et al., 2019). Here we show how to correctly reconstruct wavefields \( q_{i,n}^\pm (z_m) \) and \( p_{i,n}^\pm (z_m) \) from the recorded surface wavefield \( p_{n}^-(z_0) \) using a new concept of “inverse propagation” in 1.5D JMI.

We first consider reconstructing up-going wavefields from a surface receiver wavefield. According to equation (5), the reconstructed wavefield \( q_{i,n}^-(z_m) \) can be calculated as:

\[ q_{i,n}^-(z_m) = w(z_{m-1}, z_m)^{-1} \cdot p_{n}^-(z_{m-1}), \]

and for the reconstructed wavefield \( p_{i,n}^-(z_m) \) we have a particular relationship at the surface:

\[ p_{i,n}^-(z_0) = p_{n}^-(z_0). \]

According to equation (3), the reconstructed wavefield \( p_{i,n}^+(z_m) \) should be calculated as:

\[ p_{i,n}^+(z_m) = t^- (z_m)^{-1} \cdot q_{i,n}^+(z_m) - t^- (z_m)^{-1} \cdot r^- (z_m) \cdot p_{n}^+(z_m), \]

where the term \( p_{n}^+(z_m) \) should be from our forward simulation using an estimated source wavefield and an estimated subsurface model as the input. We call the term \( -t^- (z_m)^{-1} \cdot r^- (z_m) \cdot p_{n}^+(z_m) \) a compensation term, and it does not exist in the traditional JMI (Berkhout, 2014b; Sun et al., 2019, 2020).

At the bottom reflection interface of our model, according to equation (3), we can retrieve the inversely propagated \( p_{i,n}^+(z_{\max}) \) as:

\[ p_{i,n}^+(z_{\max}) = r^- (z_{\max})^{-1} \cdot q_{i,n}^+(z_{\max}), \]

where all other terms are 0s in equation (3) due to the fact that this is the bottom reflection interface in 1.5D FWMod. With equation (16) being available, we can further recursively reconstruct the down-going wavefields \( q_{i,n}^+(z_m) \) and \( p_{i,n}^+(z_m) \). According to equations (4) and (2), \( q_{i,n}^+(z_m) \) and \( p_{i,n}^+(z_m) \) can be calculated as:
\[ q_{i,n}^+(z_m) = w(z_{m+1}, z_m)^{-1} \cdot p_{i,n}^+(z_{m+1}), \]  
\[ p_{i,n}^+(z_m) = t^+(z_m)^{-1} \cdot q_{i,n}^+(z_m) - t^+(z_m)^{-1} \cdot r^+(z_m) \cdot p_{n-1}^-(z_m), \]

where similar to equation (15), the term \( p_{n-1}^-(z_m) \) should also be from our forward simulation. Please note that the term \( -t^+(z_m)^{-1} \cdot r^+(z_m) \cdot p_{n-1}^- \) is also a compensation term in equation (18).

Equations (13) through (18) form the complete theory for the inverse propagation of the surface receiver wavefield \( p_n^-(z_0) \) in 1.5D JMI. Different from the traditional JMI, now in our 1.5D JMI, due to the existence of compensation terms, during the process of the inverse propagation, we also need to perform another round of forward simulation using an estimated source wavefield and a user-provided subsurface model as the input.

**Example**

We use a 1.5D model shown in Figure 2(a) to demonstrate the inverse propagation in our 1.5D JMI. Here we just take the correct source wavefield and the model shown in Figure 2(a) as our estimated source wavefield and model, as our purpose in this paper is to demonstrate our ‘inverse propagation’ concept in 1.5D JMI. The maximum frequency in our source wavefield is 30 Hz, and the maximum source-receiver offset considered in our example is 5 km. We first consider the situation where only internal multiples exist, and the maximum order of multiple under consideration is 10. Using our ‘inverse propagation’ concept, the reconstructed wavefields \( p_{i,n}^\pm \) at two different depths are shown in Figures 2(b) and 2(c). Compared to the traditional JMI, it can be clearly observed that our inverse-propagation scheme accurately reconstructs those wavefields from the surface receiver wavefield \( p_n^-(z_0) \), while the traditional JMI does not yield the correct results. We further carry out a demonstration considering both surface-related and internal multiples, and we set the maximum order of multiples to 5. The corresponding results are shown in Figures 2(d) and 2(e). The advantage of our new 1.5D JMI over the traditional JMI can also be observed clearly there.

**Conclusions**

In this paper, we derive a theory framework for 1.5D FWMod and introduce a new concept of ‘inverse propagation’ in 1.5D JMI. Different from the traditional JMI, now we use subsurface velocity and density models as unknowns. In theory, our 1.5D FWMod and 1.5D JMI are capable of dealing with the AVO challenge that cannot be addressed correctly by the traditional JMI. Furthermore, with the new concept of ‘inverse propagation’, now we can correctly reconstruct subsurface wavefields from a surface receiver wavefield with all orders of multiples properly addressed. Impressive results from a synthetic model are shared, and advantages of our 1.5D JMI over the traditional JMI in reconstructing subsurface wavefields have been demonstrated. This work paves a solid way to further develop our 1.5D JMI theory.

**References**


Figure 2 (a) A 1.5D model. (b) and (c) show the reconstructed $p_{1.5D}^+$ wavefield from our 1.5D JMI and the traditional JMI (OJMI) at depth levels pointed out by the red arrows when only considering internal multiples with a maximum order of 10. (d) and (e) show the reconstructed $p_{1.5D}^+$ wavefield from our 1.5D JMI and the traditional JMI at depth levels pointed out by the red arrows when considering both surface and internal multiples with a maximum order of 5. In (b)–(e), the top-left picture shows the ground truth; the top-middle pictures shows the reconstructed wavefield by 1.5D JMI; the top-right picture shows the reconstructed wavefield by the traditional JMI; the bottom-left picture shows the true 1.5D velocity model with the red arrow pointing at the target subsurface interface; the bottom-middle picture shows the difference between the reconstructed wavefield by 1.5D JMI and the ground truth; the bottom-right picture shows the difference between the reconstructed wavefield by the traditional JMI and the ground truth.