Introduction

Spontaneous imbibition is one of the most important mechanisms of EOR in unconventional oil reservoirs with large scale hydraulic fracturing operations. Lots of attentions have been paid on SI processing in nanopores of tight matrix (Wang & Zhao, 2019), but there are lots of natural/artificial nano/micro-fractures in unconventional oil reservoirs, and studying the mechanism of spontaneous water imbibed into oil-saturated nano/micro-fractures is also of great significance for unconventional oil reservoir development. Building and solving the flow equation of water spontaneous imbibition into an oil-saturated single fracture is a key step. However, previous studies mainly focus on spontaneous imbibition in capillary tube model or pore-network model, but for spontaneous imbibition in fractures, more study should be conducted. Besides, the effect of nonwetting phase viscosity is usually ignored for simplifying the problem (Cheng et al., 2015), which may be reasonable for gas reservoir, but for oil reservoirs, the oil viscosity has great effect on spontaneous imbibition and cannot be ignored. Meanwhile, though the effect of gravity on spontaneous imbibition can be analytically solved for a gas-saturated capillary tube (Fries & Dreyer, 2008), but the analytical solution of water spontaneous imbibition into an oil-saturated single fracture with gravity effect is unreported.

In this paper, the implicit analytical model of water spontaneous imbibition into an oil-saturated fracture is derived, and verified with numerical method. The gravity of oil and water and viscosity of oil are considered in our model. Based on the fractal distribution of fractures, a core-scale spontaneous imbibition model in fractured porous media is proposed.

Mathematical model

As shown in Figure 1, the core-scale fractured porous media is assumed to be made up of a bundle of tortuous plate fractures without intersection. According to Newton’s second law, the governing equation of water spontaneous imbibition into an oil-saturated single fracture can be expressed as:

\[
\left[ \rho_w l_x + \rho_w w(l - x) \right] a = 2\sigma l \cos \theta - \frac{24 l v}{w} \left[ x \mu_w + (L - x) \mu_o \right] - \left[ \rho_o x + \rho_o (L - x) \right] w l g \sin \alpha
\]

where \( a \) is the acceleration. The \( a \) and \( v \) can be expressed with the location of oil-water interface location \( x \) as \( a = \frac{d^2 x}{d t^2} \) and \( v = \frac{d x}{d t} \). The analytical solution of imbibition velocity \( v \) is can be derived from Eq. (1) as:

\[
v = -\frac{2\sigma l \cos \theta - \left[ \rho_o x + \rho_o (L - x) \right] w l g \sin \alpha}{\frac{24 l v}{w} \left[ x \mu_w + (L - x) \mu_o \right]} \left[ \frac{24 l v}{w} \left[ x \mu_w + (L - x) \mu_o \right] f - 1 \right]
\]
when the acceleration term $a$ (inertia force) is neglected, Eq. (1) becomes:
\begin{equation}
\frac{24}{w} \left[ x_\mu_w + (L - x) \mu_o \right] \frac{dx}{dt} = 2 \sigma \cos \theta - \left[ \rho_o x + \rho_o (L - x) \right] w g \sin \alpha \tag{3}
\end{equation}

The analytical solution of $v$ can be expressed as:
\begin{equation}
v = \frac{2w \sigma \cos \theta - \left[ \rho_o x + \rho_o (L - x) \right] w^2 g \sin \alpha}{24 \left[ x_\mu_w + (L - x) \mu_o \right]} \tag{4}
\end{equation}

When the gravity of oil and water is neglected, the location of the oil-water interface $x$ can be obtained as:
\begin{equation}
x = \mu_o L - \left( \frac{\mu_o - \mu_w}{6} \right) \frac{w \sigma \cos \theta}{\sin \alpha} t \tag{5}
\end{equation}

When the gravity effect cannot be neglected, the implicit analytical solution of Eq. (3) can be derived as follows:
\begin{equation}
t = \frac{24L \mu_o}{A} \ln \left( \frac{A x + B \ln \frac{B}{A x + B}}{A x + B} \right) - \frac{24 (\mu_o - \mu_w)}{A^2} \left( A x + B \ln \frac{B}{A x + B} \right) \tag{6}
\end{equation}

where $A = -(\rho_o - \rho_o) w^2 g \sin \alpha$ and $B = 2w \sigma \cos \theta - \rho_o L w^2 g \sin \alpha$. The time for water-oil interface at the location $x$ can be obtained with Eq. (6).

There is no explicit analytical solution for $x$, and only can be solved with numerical method as:
\begin{equation}
x_i = \frac{L \mu_o}{(\mu_o - \mu_w)} \left[ \frac{1}{2} \mu_o x_{i-1}^2 + L \mu_o x_{i-1} - \frac{1}{2} \mu_o x_{i-1}^2 + \frac{w^2}{24} \left[ \frac{2 \sigma \cos \theta}{w} - \left[ \rho_o x_{i-1} + (L - x_{i-1}) \rho_o \right] g \sin \alpha \right] (t_j - t_{i-1}) \right] \tag{7}
\end{equation}

Assuming the distribution of fracture aperture can be modeled with the distribution function of $f(w)$, the total number of fractures $N$ can be obtained by:
\begin{equation}
N = \frac{\phi \tau d^2 \sin \alpha}{4 \gamma} \left[ \int_{w_{\min}}^{w_{\max}} f(w) w^2 dw \right] \tag{8}
\end{equation}

where $\gamma$ is the ratio of fracture length $l$ to fracture aperture $a$. The total oil production can be obtained by integrating the flow rate in every fracture from the maximum fracture $w_{\max}$ to minimum fracture $w_{\min}$:
\begin{equation}
Q = \int_{w_{\min}}^{w_{\max}} q(w) dN = \tau \gamma N \int_{w_{\min}}^{w_{\max}} w^2 \frac{xf(w)}{w} dw \tag{9}
\end{equation}

**Model validation and analysis**

The influences of the acceleration term $a$ and gravity on spontaneous imbibition are discussed. As Figure 2(a) shows, the acceleration term only affects the imbibition velocity at the initial stage of spontaneous imbibition, and after that the acceleration term can be ignored. Imbibition velocity increases with the increase in location $x$ due to the reduction in oil viscous force. Figure 2(b) shows that the numerical solution of water-oil interface location is identical to the implicit analytical solution, verifying the newly derived implicit analytical solution is reliable.
Figure 2 (a) The influence of the acceleration term on imbibition velocity. (b) Comparison of the water-oil interface location versus time obtained by implicit analytical model and numerical solution.

Figure 3 The influence of the gravity on spontaneous imbibition in the fracture with aperture $w=0.1\mu m$, $w=1\mu m$, $w=10\mu m$, respectively.

Figure 4 The influence of the oil-water viscosity ratio on spontaneous imbibition in the fracture with aperture $w=10\mu m$. 
Figure 3 shows the imbibition velocity without gravity is higher than that with gravity in the fracture with \( w = 10 \, \mu m \). In small fractures, such \( w = 0.1 \, \mu m \) and \( 1 \, \mu m \), the gravity effect can be ignored. Figure 4 shows the effect of oil-water viscosity ratio on imbibition velocity in the fracture with aperture \( w = 10 \, \mu m \). The imbibition velocity decreases with the oil-water viscosity ratio increasing from 1 to 5, as the viscous force which is the resistance of spontaneous imbibition increases. Figure 5 shows the gravity effect versus time in core-scale fractured porous media with different fracture distributions. As shown in Figure 5(a), assuming fracture distribution follows fractal distribution with different fractal dimension \( D_f \) and fracture apertures vary from \( w_{\text{min}} = 0.01 \, \mu m \) to \( w_{\text{max}} = 10 \, \mu m \), the core-scale oil recovery rate versus time can be calculated, as shown in Figure 5(b). With \( D_f \) increasing from 1.2 to 1.8, the core-scale imbibition rate decreases significantly due to the reduction proportion of large pores.

![Figure 5](image)

**Figure 5** The oil recovery rate versus time in core-scale fractured porous media with different fracture distributions.

Conclusions

A new implicit analytical model of water spontaneous imbibition into an oil-saturated fracture with the effects of gravity and nonwetting phase viscosity is derived, and a spontaneous imbibition model in core-scale fractured model is proposed with the given fracture distribution. The results show that acceleration term (inertia force) only affects imbibition rate at the initial stage of imbibition and can be ignored. For the large fractures, such as fracture aperture greater than \( 10 \, \mu m \), the gravity effect cannot be ignored. The imbibition velocity increases with the time due to the reduction in oil viscous force, and the imbibition rate in core-scale fractured porous media is more dominated by fracture aperture distribution.

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References

