Introduction

When a seismic wave propagates through viscous media such as unconsolidated soil and semi-consolidated rocks, absorption attenuation can change the velocities, frequencies, amplitudes, and phases of the seismic wave field, affecting the spatial, and temporal resolution of the migration images (e.g., Schuster et al., 2017). However, these effects are not yet clear for limited-scope and discrete-sampled seismic data in complex media, particularly when attenuation compensation is considered in migration. Focal beam analysis (Berkhout et al., 2001) is a method that directly relates prestack migration theory to the assessment of seismic acquisition geometries, providing a direct link between the acquisition parameters at the surface, and image quality at a target location in the subsurface. However, most of these studies have only treated the earth model as a lossless acoustic/elastic medium and ignored the effects of absorption attenuation in decreasing spatial resolution. In this study, we extend the focal beam method to viscous media to study the relationship between imaging resolution and acquisition geometry, which can be used to quantify the performance of acquisition geometries based on spatial resolutions and amplitudes. This article is organized as follows: In the methods section, we extend the method of focal beam analysis to a viscous medium, in which the viscoacoustic Fourier pseudo spectral method (PSM) (e.g., Carcione, 2010) is used in both forward and backward wave extrapolations. In the results section, we determine the resolution and amplitude of migration images using four schemes: 1) acoustic modeling followed by acoustic migration, 2) viscoacoustic modeling followed by acoustic migration without applying Q compensation, 3) viscoacoustic modeling followed by viscoacoustic migration applying accurate Q compensation, and 4) viscoacoustic modeling followed by viscoacoustic migration applying inaccurately evaluated Q compensation. We apply our method to a complex velocity and Q model to determine the effects of absorption attenuation on the choice of sampling lengths from the perspectives of spatial resolution and amplitude. Lastly, the conclusion section provides the major findings of the study.

Methods

The focal beam analysis method (Berkhout et al., 2001) originated from migration imaging using the double-focusing concept as explained using the WRW model, which describes data of a seismic survey with the aid of a data matrix \( P(z_d, z_s) \) in the frequency domain. In this model, each element of this matrix represents a monochromatic component of the data recorded by a detector at depth level \( z_d \) generated by a source at depth level \( z_s \). In a viscous medium, the attenuation effects should be incorporated in the upward and downward propagation matrices \( W^- \) and \( W^+ \); if we set the detectors and the sources at the same surface level \( z_0 \), the monochromatic component of the \( k \)th grid-point response related to depth level \( z_m \) can be described as follows:

\[
\delta_k P(z_0, z_0) = D(z_0) W_Q(z_0, z_m) \delta_k R(z_m, z_m) W_Q^+(z_m, z_0) S(z_0),
\]

where \( D(z_0) \) is the detector matrix, \( S(z_0) \) is the source matrix, \( \delta_k R(z_m, z_m) \) is the grid-point reflectivity matrix containing subsurface information, \( W_Q(z_0, z_m) \) is a forward wave propagation matrix describing upward wave propagation from \( z_m \) to the surface level \( z_0 \), and \( W_Q^+(z_m, z_0) \) is also a forward wave propagation matrix describing downward wave propagation from the surface level \( z_0 \) to \( z_m \), the subscript \( Q \) indicates the seismic wave propagation in the viscous medium.

The objective of the migration is to image the subsurface in terms of reflectivity. Hence, the wave propagation effect has to be removed from the data. In terms of the WRW model, this means that the terms \( W_Q(z_0, z_m) \) and \( W_Q^+(z_m, z_0) \) have to be removed, implying that the attenuation effects also need to be removed, such that we need to complete a \( Q \) compensation for the attenuated seismic data to mitigate the amplitude decay and phase dispersion effects. This is achieved by adding two focusing operators, a detector-focusing operator \( F_Q(z_m, z_0) \) and source-focusing operator \( F_Q(z_0, z_m) \). For focusing on both the detection and source sides, the results can be written as follows:

\[
\delta_k P_{DQ}(z_m, z_m) = F_Q(z_m, z_0) D(z_0) W_Q(z_0, z_m) \delta_k R(z_m, z_m) W_Q^+(z_m, z_0) S(z_0) F_Q^+(z_0, z_m),
\]

where \( DQ \) (double-side \( Q \)) represents the attenuation effects considered during both the propagation and the focusing processes. This process is referred to as double-focusing. In Eq. (1), by applying the
focusing operators $\mathbf{F}_Q^+(z_n, z_0)$ and $\mathbf{F}_Q^-(z_0, z_n)$ to a depth $z_n$ besides the target depth $z_m$, the resolution function can be expressed as follows:

$$\delta_k \mathbf{P}_{DQ}(z_n, z_m, z_m, z_n) = \mathbf{F}_Q^+(z_n, z_0) \delta_k \mathbf{P}(z_0, z_0) \mathbf{F}_Q^-(z_0, z_n) = \left[ \mathbf{F}_Q^+(z_n, z_0) \mathbf{D}(z_0) \mathbf{W}_Q^-(z_0, z_m) \right] \delta_k \mathbf{R}(z_m, z_m) \left[ \mathbf{W}_Q^+(z_m, z_0) \mathbf{S}(z_0) \mathbf{F}_Q^-(z_0, z_n) \right].$$

(3)

$\delta_k \mathbf{P}_{DQ}(z_n, z_m, z_m, z_n)$ represent the double-focusing matrix from the target point level $z_m$ to the focused depth level $z_n$. In Eq. (3), the same accurate $Q$ model is applied in the migration as in the modeling. When the accurate $Q$ model cannot be provided, it can be evaluated based on the empirical relations to the velocity model (e.g., Li, 1993) as follows:

$$Q = 14 \nu^{-2},$$

(4)

where $\nu$ is the P-wave velocity (km/s). Then, the double-focusing process can be rewritten as follows:

$$\delta_k \mathbf{P}_{EQ}(z_n, z_m, z_m, z_n) = \left[ \mathbf{F}_E^+(z_n, z_0) \mathbf{D}(z_0) \mathbf{W}_Q^-(z_0, z_m) \right] \delta_k \mathbf{R}(z_m, z_m) \times \left[ \mathbf{W}_Q^+(z_m, z_0) \mathbf{S}(z_0) \mathbf{F}_{EQ}^-(z_0, z_n) \right].$$

(5)

Here, the subscript $EQ$ means the evaluated $Q$ model is used in the focusing matrix $\mathbf{F}_{EQ}$. If we do not consider the $Q$ compensation in the focusing matrix $\mathbf{F}$, the double-focusing equation can be written as follows:

$$\delta_k \mathbf{P}_{SQ}(z_n, z_m, z_m, z_n) = \left[ \mathbf{F}_S^+(z_n, z_0) \mathbf{D}(z_0) \mathbf{W}_Q^-(z_0, z_m) \right] \delta_k \mathbf{R}(z_m, z_m) \times \left[ \mathbf{W}_Q^+(z_m, z_0) \mathbf{S}(z_0) \mathbf{F}_S^-(z_0, z_n) \right].$$

(6)

The subscript $SQ$ represents the single-side effect of $Q$ attenuation considered in the propagation matrix $\mathbf{W}$. Similarly, if we do not consider $Q$ attenuation effects in both the propagation matrix $\mathbf{W}$ and focusing matrix $\mathbf{F}$, the double-focusing equation will be written as follows:

$$\delta_k \mathbf{P}_{NQ}(z_n, z_m, z_m, z_n) = \left[ \mathbf{F}_N^+(z_n, z_0) \mathbf{D}(z_0) \mathbf{W}_N^-(z_0, z_m) \right] \delta_k \mathbf{R}(z_m, z_m) \times \left[ \mathbf{W}_N^+(z_m, z_0) \mathbf{S}(z_0) \mathbf{F}_N^-(z_0, z_n) \right].$$

(7)

The subscript $NQ$ represents no $Q$ attenuation is considered. The double-focusing results in Eqs. (3), (5), (6), and (7) represent the resolution function produced by different $Q$-compensated migrations in different media. In Eq. (7), $\delta_k \mathbf{P}_{NQ}(z_n, z_m, z_m, z_n)$ represents the ideal resolution of the conventional focal beam analysis in lossless media. In Eq. (3), $\delta_k \mathbf{P}_{DQ}(z_n, z_m, z_m, z_n)$ represents the resolution function of the $Q$-compensated migration with an accurate $Q$ model in lossy media. In Eq. (5), $\delta_k \mathbf{P}_{EQ}(z_n, z_m, z_m, z_n)$ represents the resolution function of the $Q$-compensated migration with an inaccurately evaluated $Q$ model in lossy media. In Eq. (6), $\delta_k \mathbf{P}_{SQ}(z_n, z_m, z_m, z_n)$ represents the resolution function of the conventional $Q$-uncompensated migration in lossy media.

In the implementation of Eqs. (3), (5), (6), and (7), all resolution functions contain forward wavefield extrapolation terms ($\mathbf{W}$) and reverse wavefield extrapolation terms ($\mathbf{F}$). The efficiency and accuracy of the focal beam analysis greatly depends on the choice of the wave propagation method. In viscous media, we need to use viscoacoustic modeling to simulate wave propagation as accurately as possible. Considering the trade-off between computing efficiency and accuracy, we choose to use the fractional Fourier PSM (e.g., Carcione et al., 2010) to perform forward wavefield extrapolation in viscous media, which can decrease the storage requirements of intermediate wavefields to near that in lossless media. On the other hand, imaging resolutions also greatly depend on the manner, by which the prestack migration compensates the $Q$ attenuations. Here, we use two migration (backward-focusing) strategies to implement focal beam analysis: 1) conventional reverse time migration (RTM) without considering $Q$ compensation (e.g., Baysal et al., 1983) and 2) $Q$-compensated reverse time migration ($Q$-RTM) (e.g., Li et al., 2019). The procedure of focal beam analysis in viscous media is shown in Fig. 1.

**Example**

We use a section of the BP gas model (Billette and Brandberg-Dahl, 2004) characterized by a gas chimney in the central-top low-velocity and high-attenuation zone. Fig. 2 shows the velocity model, accurate $Q$ model, and evaluated $Q$ model calculated using Eq. (4). The model size is $1201 \times 321$ with a grid spacing of $dx = dz = 10$ m. The source is a Ricker wavelet with a peak frequency of 20 Hz and the recording length is 2 s with a temporal sampling interval of 1 ms. We assumed that the angle-independent point source is located at the point $(x, z) = (6000, 2500)$ m and the receivers are located at a depth of 50 m. The detector-line lengths range from 4000 to 12,000 m, increasing 2000 m each time.
Fig. 3 shows the resultant horizontal resolution $R_x$, vertical resolution $R_z$, and normalized peak amplitude $A_p$ for the layered model, which are measured from the resolution functions calculated using Eqs. (3), (5), (6), and (7). As shown in Fig. 3a, all the horizontal resolutions $R_x^Q$, $R_x^S$, $R_x^{DQ}$, and $R_x^{EQ}$ improve with increased detector sampling length until arriving at their limits. $R_z^S$ in the viscoacoustic modeling shows a much lower horizontal resolution limit than that of $R_z^Q$ in the acoustic modeling. By applying accurate amplitude compensations, $Q$-RTM migration nearly recovers the resolution of $R_z^{DQ}$ to the same level as that of $R_z^Q$. When the evaluated inaccurate $Q$ model is used, $Q$-RTM can also partly recover the resolution of $R_z^Q$. In Fig. 3b, $R_z^{Q}, R_z^S, R_z^{DQ},$ and $R_z^{EQ}$ represent the vertical resolutions of Eqs. (3), (5), (6), and (7), respectively, which only slightly change with the detector sampling length, indicating that the sampling length has a limited influence on the vertical resolution. By comparison, $R_z^{Q}$ shows the highest vertical resolution limit, closely followed by $R_z^{DQ}$ and $R_z^{EQ}$, while $R_z^S$ shows the obviously lowest. In Fig. 3c, $A_p^Q$, $A_p^{DQ}$, $A_p^{EQ}$, and $A_p^S$ are the normalized peak amplitudes. Here, $A_p^Q$ produces the highest peak amplitudes closely followed by $A_p^{DQ}$ and $A_p^{EQ}$, while $A_p^S$ shows the lowest values. In this case, from both the perspectives of resolution and amplitude, we can draw a conclusion that it is necessary to perform an amplitude compensation (e.g., using $Q$-RTM) during (or before) migration for viscoacoustic media. Furthermore, we should also consider that $Q$ compensation during migration cannot totally eliminate the effects produced by the absorption attenuation in wave propagation, particularly when an accurate $Q$ model is unknown. In areas of strong absorption attenuation (with a small $Q$), whether applying $Q$-RTM or not, a longer detector length is needed to achieve the same resolution, and amplitude as that in areas of weak absorption attenuation (with a large $Q$). The higher the expected resolution, the larger the sampling length increment.

Conclusions

In this study, we extend the focal beam method to viscous media to study the relationship between imaging resolution and acquisition geometry, which can be used to quantify the performance of acquisition geometries in viscous media. We determine the resolution and amplitude of migration images using four schemes: 1) acoustic modeling followed by acoustic migration, 2) viscoacoustic modeling followed by acoustic migration without applying $Q$ compensation, 3) viscoacoustic modeling followed by viscoacoustic migration applying accurate $Q$ compensation, and 4) viscoacoustic modeling followed by viscoacoustic migration applying inaccurately evaluated $Q$ compensation. The results show it is necessary to perform an amplitude compensation (e.g., using $Q$-RTM) during (or before) migration for viscoacoustic media from both perspectives of improving resolution and recovering amplitude. However, $Q$ compensation during migration cannot totally eliminate the effects produced by absorption attenuation in wave propagation, particularly when an accurate $Q$ model is unknown. In areas of strong absorption attenuation (with a small $Q$), without accurate $Q$ compensation, a longer detector length is needed to achieve the same resolution and amplitude as that in areas of weak absorption attenuation (with a large $Q$). The higher the expected resolution, the larger the sampling length increment.

Reference


**Figure 1.** Workflow for focal beam analysis in viscous media.

**Figure 2.** (a) Velocity and (b) Q of the BP gas model. (c) Estimated Q model calculated using the velocity model. The blue zone at 1000-m depth in the Q model is a high-attenuation gas zone.

**Figure 3.** (a) Horizontal resolution, (b) vertical resolution, and (c) normalized peak amplitude calculated using Eqs. (3), (5), (6), and (7) at different detector sampling lengths. The solid line represents the RTM results of the acoustic modeling data, the dashed line represents the RTM results of the viscoacoustic modeling data, the densely dotted line represents the Q-RTM results using an accurate Q model, and the loosely dotted line represents the Q-RTM results using an evaluated Q model.