Introduction

Prestack seismic inversion is a multimodal problem due to the nonlinearity of physical model relating seismic data to elastic parameters. To address the problem, global optimisation algorithm (Sen and Stoffam, 2013) possesses strong exploration capability by taking advantage of stochastic searching mechanism. In particular, particle swarm optimisation (PSO) has been successively employed to handling both pre- and post-stack seismic inversion tasks (Guo et al., 2018). Although PSO is easily implemented and fast converged, it risks premature convergence due to undesirable guidance particles. Besides, prestack inversion is also a multiparameter problem, leading to the instabilities of multiple results. As a type of stochastic algorithm, PSO usually fails to promote the stability of multiple results due to the random walks existed in the model update.

Among different variants of PSO, orthogonal particle swarm optimisation (OPSO) (Ho et al., 2008) utilizes orthogonal learning (OL) strategy, aiming to explore and preserve effective information of particle positions. In specific, orthogonal experimental design (OED) is capable of formulating the best combination of a particle (potential solution) through the whole dimensions of the problem, which leads to the orthogonal learning PSO (OLPSO) (Zhan et al., 2011). Considering that solution to prestack seismic inversion consists of multiple dimensions for each particle, if multiple dimensional information can be orthogonal learned during model update, the multiple parameters can be thereby correlated and stabilized. Therefore, the combination mechanism offered by OED in OLPSO provides a promising approach for realizing stabilized multiple results.

In this abstract, a stabilized prestack seismic inversion method using orthogonal learning hybrid particle swarm optimisation (OLHPSO) is proposed, in which the OED-based OL strategy and the Metropolis-based probabilistic mechanism are incorporated into PSO. The advantages of the proposed method are featured as: 1) the multiple results are correlated and stabilized during model update by the OED-based OL strategy in OLPSO algorithm, and 2) the Metropolis-based probabilistic mechanism is employed to the selection of the guidance particle, which improves the exploration capability of OLPSO in solving highly nonlinear inversion problem.

Theory

Objective function

Given the forward operator $G$ and the elastic parameters $m$ representing properties of subsurface layers, the seismic responses can be simulated as

$$d = G(m) + e$$

where $d$ is the recorded seismic data, $G$ is the forward operator, by which seismic responses are simulated by convolving reflectivity series calculated by the Zoeppritz equation with source wavelets, and $e$ is the random noise. The goal of seismic inversion is to estimate a series of elastic parameters, minimizing the bias between simulated and observed data. The objective function is built based on the Bayesian theory as (Buland and Omre, 2003)

$$P(m | d) = \frac{P(d | m) \times P(m)}{P(d)} \propto P(d | m) \times P(m)$$

where $P(m | d)$ is the posterior probability function, $P(d | m)$ is the likelihood function, and $P(m)$ is the prior probability function. The estimation of unknown parameters $m$ can be considered as a MAP (maximum a posteriori) problem, which can be realized by optimising the objective function as

$$\hat{m} = \arg \min m \left( \frac{1}{2\sigma} \| d - G(m) \|^2 + \lambda (m - \bar{m})^T C_m^{-1} (m - \bar{m}) \right)$$

where $\sigma$ is the standard deviation of the random noise, $C_m$ and $\bar{m}$ are the covariance matrices and the mean values of $m$ derived from the logging data, and $\lambda$ is the trade-off parameter.

Particle Swarm optimisation

To initialize standard PSO procedure, a population array of $n$ particles in the D-dimension is defined, and each particle denotes a potential solution to the problem. Particle $i$ has a vector of positions $m_i$ and velocities $v_i$. The personal best position and the global best position in D-dimension are determined.
based on the fitness value. The two guidance particles are used to lead the update of each particle’s position and velocity. The velocity of the $i$ particle at the $k$ generation (iteration) is updated by

$$v_{i,D}^k = \omega v_{i,D}^{k-1} + c_1 r_1 (p_{best_D} - m_{i,D}^k) + c_2 r_2 (g_{best_D} - m_{i,D}^k)$$

(4)

where $m_{i,D}^k$ and $m_{i,D}^{best}$ are the personal and global best positions, respectively, $\omega$ is the inertia weight, $c_1$ and $c_2$ are acceleration coefficients, and $r_1$ and $r_2$ are two independently random numbers between $[0, 1]$. Then, the new position of the $i$ particle at the $k$ generation is obtained by

$$m_{i,D}^k = m_{i,D}^{k-1} + v_{i,D}^k$$

(5)

The final results are achieved iteratively by equations (4) and (5) when each particle has the same $m_{i,D}^{best}$ as $m_{i,D}^{best}$.

Orthogonal Learning Hybrid Particle Swarm optimisation

In traditional PSO algorithm, the update of particle positions is controlled by two guidance particles. If the two guidance particles are in opposite direction, the update of particles may lead to oscillation. Besides, if one guidance particle has good values on certain dimension of a solution vector while bad values on other dimensions, the results might be unexpectedly deteriorated. Prestack seismic inversion is a multimodal and multiparameter optimisation problem, the problems mentioned above will cause premature convergence and instabilities of multiple results if being solved by traditional PSO.

Orthogonal experimental design (OED) is a mathematical tool to analyze the effect of multi-factors and multi-level problems, which makes it possible for different variables to learn from each other and assist formulating a best combination of a potential solution. Aided by the OED strategy, Zhan et al. (2011) proposed the OLPSO algorithm, in which the guidance particle is learned from the personal and global best particles. Based on equation (4), the model update in OLPSO is modified as

$$v_{i,D}^k = \omega v_{i,D}^{k-1} + c r (m_{D}^{OED} - m_{i,D}^{k})$$

(6)

and

$$m_{D}^{OED} = m_{D}^{best} \otimes m_{i,D}^{best}$$

(7)

where $\otimes$ denotes the OED operator, which can be referred to the Appendix of Zhan et al. (2011).

In traditional OLPSO, the best combination by OED is undoubtedly accepted as the guidance particle. To enhance the searching capability and avoid premature convergence of the algorithm, the probabilistic mechanism inspired from Metropolis criterion (Sen and Stoffam, 2013) is introduced to controlling the acceptance of the OED-based combination, i.e.,

$$P_{AC}(C_i,T) = \frac{\exp \left( -\frac{F(C_i) - F(C_{OED})}{T} \right)}{\sum_{i=1}^{T} \exp \left( -\frac{F(C_i) - F(C_{OED})}{T} \right)}$$

(8)

where $F$ is the misfit function, $C_i$ is the $i$ combination and $C_{OED}$ is the OED-based combination. For each particle, after calculating $P_{AC}$ of the all combinations, the roulette wheel selection is used to determine which one is accepted as the guidance particle at that iteration. Based on equation (8), the probability of accepting $C_i$ is inversely proportional to the difference between itself and $C_{OED}$ at the given $T$. As the $T$ anneals iteratively, other combinations may have a certain probability to be accepted at early iteration, while such probability decreases gradually towards zero at the end of iteration (the algorithm turns out to be traditional OLPSO).

Synthetic data test

The data from Marmousi model is extracted to perform a synthetic test. Figure 1a–b shows the results using PSO, OLPSO, and OLHPSO, respectively. Comparing the PSO (Figure 1a) and OLPSO (Figure 1b) results, the latter exhibit better agreement with the true models. Furthermore, the OLHPSO results (Figure 1c) removes more noises and realizes better agreement with the true models compared with OLPSO results, especially for 650–700 ms for the $V_S$ and the density.

Since the algorithm involves random walks, we evaluate the methods for a degree of uncertainty by performing them sequentially with independent tests. Figure 2a–c shows the scattering plots of the 100 results for the 600–640 ms section, overlapped by the estimated distribution probabilities between $V_P$ and $V_S$. $V_P$ and $V_S$ density. It shows that the OLHPSO results (Figure 2c) show smaller distribution
ellipses, indicating the OLHPSO results have less uncertainty and the multiple results are more stable since they exhibit fewer deviation or bias from the true values. Figure 2d shows the evolution of fitness value during the inversion process, indicating that the convergence performance of OLHPSO is better than the others.

Figure 1. The results (red) of $V_P$, $V_S$, and density using (a) PSO, (b) OLPSO, and (c) the proposed OLHPSO, overlapped by the true models (blue).

Figure 2. Scattering results using (a) PSO, (b) OLPSO, and (c) the proposed OLHPSO for $V_P$ versus $V_S$ (top row) and $V_P$ versus density (bottom row), respectively, with the estimated distribution probabilities (the white points indicate the 100 independent results and the black dots indicate the true values), and (d) the fitness (misfit) evolution as a function of iteration.

**Field data application**

A practical application is presented to validate the proposed method. The field data set are acquired from the Pearl River Delta in the North China Sea for offshore exploration of carbonate reservoirs. An arbitrary two-dimension section is chosen from the data set for the analysis. The well log penetrated the section is located at the 301st CDP. The section contains 701 angle gathers (CDPs), where each gather has 12 traces ranging from 3° to 36° with 3° interval, and each trace has 167 samples with 2 ms sampling rate. The study area shows complex geological condition and the lithologies associated with the potential reservoir show apparent variance, which are identified as sandstone, limestone (target layer), and shale from top to bottom based on log profile. In order to address the lithologic variation in this area, three different prior terms are estimated, each with specified covariance matrices and mean-values of elastic parameters for the corresponding layer (lithology).

Comparing the PSO and OLPSO results, respectively, the latter show improved stability with less anomalies, especially for the $V_P$ (Figure 3b) and the density (Figure 5b). However, no obvious improvement is observed for the $V_S$ (Figure 4b). Further comparing the results using the proposed OLHPSO with the others, the proposed method not only enhance the stability of multiple parameters but also improve the convergence performance, since the horizontal continuity of the top target layer is the best, especially for the $V_P$ (indicated by the arrows in Figure 3c). Besides, a closer view of the $V_S$ indicates the fewest noises within the target layer (indicated by the arrows in Figure 4c).
Figure 3. The results of $V_p$ using (a) PSO, (b) OLPSO, and (c) the proposed OLHPSO.

Figure 4. The results of $V_s$ using (a) PSO, (b) OLPSO, and (c) the proposed OLHPSO.

Figure 5. The results of density using (a) PSO, (b) OLPSO, and (c) the proposed OLHPSO.

Conclusions

In this abstract, we propose the orthogonal learning hybrid particle swarm optimisation (OLHPSO) algorithm and incorporate it into the three-term prestack seismic inversion based on Bayesian framework, aiming to improve the stability of multiple results and alleviate the premature convergence of the algorithm. In specific, the OLHPSO takes the advantages of the OED-based OL strategy and the Metropolis-based probabilistic mechanism, with special attention of stabilizing multiple results during model update as well as improving convergence performance. Synthetic tests indicate that the multiple results using OLHPSO exhibit better agreement with true models, and they are more stable with less uncertainty, compared to those using PSO or OLPSO. Field data application further demonstrates the effectiveness of the proposed method, since the OLHPSO results show improved stability with less anomalies and reveal the top target layer with better lateral continuity, which assist for fine evaluation of potential reservoirs under complex geological condition.

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References