Nonparametric estimation method for array acoustic dispersion information

Introduction

Dispersion information in array acoustic data is of great significance for reservoir evaluation, among which nonparametric methods are widely used because they do not need prior information of the number of mode waves. Capon (1969) proposed a spectral estimation method based on adaptive filtering to analyze the frequency-wavenumber spectrum estimation of array signals. Li and Stoica (1996) proposed a filter design method considering the interference of other modes, which is known as the Amplitude and Phase Estimation method (APES). The Weighted Spectral Semblance (WSS) method proposed by Nolte (1997) is an application based on the Fourier transform theory.

There is no nonparametric algorithm that can simultaneously combine good adaptability, noise immunity, and resolution. This paper analyzes and compares the performance of the above algorithms more systematically from both simulated and measured data, which can provide some reference for audiences with different needs.

Method

Capon et al. proposed to minimize the array output power on the premise of ensuring a certain response to the desired signal direction, forming optimization problems:

$$\min_w w^H R_w s.t. \ w^H a = 1 \quad (1)$$

where \(w\) is the filter weight coefficient and \(R\) is the signal covariance matrix. By using the Lagrange multiplier method to get the optimal solution: \(w = R^{-1}a / a^HR^{-1}a\), so that the complex amplitude estimation can be obtained.

APES and FB-APES are designed to optimize the filter by adaptively estimating the covariance matrix of the noise in the signal so that only the sinusoidal components containing specific components pass through the filter without loss, which suppress the output of other sinusoidal components and noise at the same time, thus can be used to estimate the amplitude spectrum of the signal. The optimization constraint is given by:

$$\min_w w^H Q_w s.t. \ w^H a = 1 \quad (2)$$

where \(Q\) can be considered as the noise covariance matrix.

The Fourier transform method and its variants can be regarded as filtering data with frequency-dependent narrowband filters and estimating the power at the output of the filters. The implementation of WSS is mainly in two steps: spectrum coherence and frequency domain weighting. For each frequency \(\omega\), we now define a vector:

$$D(\omega) = \begin{bmatrix} D_1(\omega) & D_2(\omega) & \cdots & D_n(\omega) \end{bmatrix}^T \quad (3)$$

The elements of \(D(\omega)\) are the Fourier coefficients of the \(n\) receivers at frequency \(\omega\). We now define a vector \(S(\omega, p_z)\) whose elements are the phase shifts of the receivers relative to the first one.

$$S(\omega, p_z) = [1 \ \exp(-i\omega p_z \Delta z) \ \cdots \ \exp(-i\omega p_z (n-1) \Delta z)]^T \quad (4)$$

where \(\Delta z\) denotes the receiver spacing and \(p_z\) denotes the slowness of this mode. Next, we define a fitness function \(f(\omega, p_z)\) which is the absolute value of the normalized correlation of the vectors \(D(\omega)\) and \(S(\omega, p_z)\).

$$f(\omega, p_z) = \frac{|D^*(\omega)S(\omega, p_z)|}{\sqrt{D^*(\omega)D(\omega)S^*(\omega, p_z)S(\omega, p_z)}} \quad (5)$$
The molecular part of the fitness function is FTM. First, we resample our data in the frequency domain (by padding the time-domain traces with zeros). We then obtain a denser sampling in the frequency domain. We denote the frequency points in this new sampling by \( \omega_k \). Then, instead of maximizing \( f(\omega, p_z) \), we maximize

\[
F(\omega_k, p_z) = \sum_{j=k-l}^{k+l} W(\omega_j) f(\omega_j, p_z)
\]

where \( W(\omega_j) \) is a Gaussian weight function:

\[
W(\omega_j) = \exp\left(-\frac{(\omega_k - \omega_j)^2}{2\sigma^2}\right)
\]

Where \( \sigma = \frac{N_w \Delta \omega}{w} \), \( N_w \) denotes the number of spectral points involved in the weighting, and \( \Delta \omega \) represents the frequency interval.

**Numerical simulation experiment**

According to the analytical solution of array acoustic waveform signals, single frequency signals of three-mode waves with the slowness of 50 us/ft, 60 us/ft, and 90 us/ft are designed, and the processing results are shown in Figure 1. By using the high-order expansion technique, it proves that Capon is biased downward, while APES is unbiased, and the error of FB-Capon is about half that of Capon (H. Li, 1998). The WSS method cannot effectively distinguish the true slowness of adjacent mode waves when their slowness intervals are small. When the analog signal is a single frequency signal, the WSS method cannot be weighted by multiple frequency points and thus becomes a single frequency point superposition, and its noise immunity is comparable to that of the FTM; in the actual data processing, the noise immunity can be improved after weighting by multiple frequency points. The random white Gaussian noise from 0dB to 50dB at intervals of 0.05dB is added to a single frequency (8kHz) single-mode (80us/ft) signal, and the comparison diagram of anti-noise ability is obtained by counting the relative error of slowness estimation, which are shown in Figure 2. The noise immunity is improved by using forward-backward data simultaneously compared to the forward-only data processing method.

**Figure 1** Processing results of simulation data by the nonparametric method. Black markers indicate the slowness and amplitude of actual modes. The red markers are the peak points of the curve, which represent the estimated amplitude and slowness. The number next to the mark corresponds to the serial number of the method in the legend.
**Figure 2** Comparison of anti-noise performance of nonparametric methods. The filter length of Capon, APES, and their forward and backward algorithms is 6, and the total number of analog data receivers is 13. Each data point is the average result of 1000 experiments.

**Example of actual data**

The actual data is the array acoustic monopole data collected by the XMAC instrument of Atlas Company, which is collected from Liaohe Oilfield in northeast China.

The filter lengths of APES, Capon, and their forward and backward variants have a great influence on the algorithm resolution (See Figure 3). With the increase of length, the resolution of APES first increases and then decreases, showing a symmetrical change trend. When the length of APES and FB-APES filters is N/2, the resolution and stability are better (N is the total number of receivers). The resolution of the Capon beamforming algorithm increases with the increase of filter length, and the resolution of Capon and FB-Capon is better at a higher order. The stability of the algorithm decreases with the increase of the filter length, which can be optimized by the diagonal loading technique.

Comparing the processing results with the highest resolution among the six methods selected from a certain underground interval, the obtained results are shown in Figure 4.

**Conclusion**

Both FTM and WSS share the same kind of disadvantage in large side lobe, distorted amplitude slowness estimation, and low resolution, but they have strong anti-noise ability and high computational efficiency due to their algorithm. APES and FB-APES can accurately estimate amplitude, phase, and slowness, with small sidelobe and high resolution, yet weak in anti-noise ability. Capon and FB-Capon have high resolution, strong anti-noise ability, and accurate slowness estimation, but cannot accurately estimate the signal amplitude and phase. Using forward and backward data can improve the anti-noise ability of the algorithm and reduce the estimation error, but the complexity of the algorithm increases.

**References**


**Figure 3** Comparison of resolutions of different filter lengths ($M$): (a) Results of APES. (b) Results of FB-APES. (c) Results of Capon. (d) Results of FB-Capon.

**Figure 4** Comparison of maximum resolution of processing results of the Stoneley wave nonparametric method in a certain layer of actual data. APES and FB-APES curves are the processing results of filter length 4, Capon curve filter length 7, and FB-Capon curve filter length 8.