Introduction

One of the goals of reservoir simulation is seeking optimal production schemes that maximize oil and gas production. Usually, the reservoir engineer achieves this goal by trial and error to determine the adjustable parameter and their combinations. Optimisation of oil production involves maximizing or minimizing the objective function(s) subjected to a set of manipulated control variables. In waterflood optimisation, the objective function could include maximization of oil production, minimization of water production or delayed water breakthrough which is subject to the set of controls such as bottom-hole pressures (BHPs), flow rates and choke sizes.

Several optimisation algorithms have been developed and applied to reservoir development problems especially waterflooding. These algorithms assume that both the technical and operational parameters are deterministically known, and this is responsible for poor performance when implemented on the field. When modelling and optimising a dynamic process, it is, therefore, expedient to model uncertainties associated with the process in a robust optimization. This is because it is impractical to adequately capture all the dynamics and properties of the process.

Method

The Markowitz classical theory is an approach for carrying out optimisation under uncertainty (Markovitz, 1952). Research by Couët, et al. (2000) has adapted this theory for its application in oil and gas field operations using an “efficient frontier” approach and a corresponding decision table. An efficient frontier is utilised to qualify the relationship between the optimized mean value of the performance metric and its corresponding standard deviation. Each data point on the efficient frontier represents a trade-off between the maximisation of the expected result (i.e. the mean) and the minimisation of the uncertainty (standard deviation). A detailed explanation of this optimisation technique can be found in Bailey & Couet, (2005).

In this study, the Response Surface Methodology of the Design of Experiment (DoE) was used to create proxy models that approximated the reservoir model. DoE has already received numerous applications in the petroleum industry particularly in the area of uncertainty screening and quantification. It significantly decreases the number of simulations required to assess uncertainties.

Eleven uncertain geological variables were identified to affected oil production from the reservoir. They include oil viscosity, porosity, vertical permeability, horizontal permeability, connate water saturation, fault transmissibilities, aquifer pore volume and directional transmissibilities across the reservoir grid. Constructing a surrogate model for an optimisation problem considering fourteen parameters (three engineering and eleven geological variables) would incur high computational costs. Therefore, a preliminary dynamic sensitivity analysis was carried out using Plackett-Burman experimental design coupled with Monte Carlo simulation (MCS) to determine how they affect the cumulative oil production response. The parameters which significantly influenced the response (heavy hitters) were selected and employed in constructing the surrogate model.

The training and validation experimental data sets were designed using the Box-Behnken experimental design. A polynomial regression proxy model was used to correlate the engineering control variables (the water injection rate and oil production rate from wells in two groups) and geological uncertainties to the field total oil production, FOPT. Robust optimisation was then performed using the surrogate model instead of full reservoir simulations. Mean-variance graphs were then constructed for two different scenarios. They were optimisation involving one uncertainty (vertical permeability), and that involving four uncertain geological parameters (vertical permeabilities, permeability in y-direction, porosity, and viscosity). These parameters were identified to have the most dominant effect on cumulative oil production based on the sensitivity analysis described in the previous paragraph. A genetic algorithm was applied in the optimisation routine.

Generally, in mean-variance optimisation problems involving \( m \) uncertainty parameters, \( N = m^n \) (where \( n \) = number of sample points) reservoir model realisations require a limited number of sample points which can lead to inaccurate estimation of mean and standard deviation, and by extension, the objective function. For example, the actual distribution of the uncertain parameter is ignored and its low, median and high values (with equal probability) are used to represent the distribution. In this
research, it was discovered that at least 100 sample points (realisations) were required to adequately capture the distribution of the uncertain parameters and perform optimisation under uncertainty.

Finally, to depict the different levels of optimality of the solutions obtained from the GA optimisation, a non-dominated sorting algorithm like that of the NSGA was applied to rank the solutions from the optimisation routine involving four uncertainties. This was done because the optimisation of the objective function creates a “pseudo” bi-objective optimisation problem in which the objectives are to maximize the mean, $\mu$ and minimise the risk or standard deviation, $\sigma$ of the objective function. Non-dominated sorting was not performed for the optimisation case involving the most dominant uncertainty (one uncertainty) because there was an absence of a distinct efficient frontier due similarity of the optimal solutions for the various risk-aversion factors. Summarily, the methodology applied in this study is shown in Figure 1.

Results

Figures 2a and 2b present the efficient frontiers of the robust scenarios considering one and four uncertain variable(s) respectively. The five large markers represent the optimum cumulative oil production for risk-aversion factors of 0.0, 1.0, 2.0, 3.0 and 4.0 respectively.

In Figure 2a, a cluster of the optimal solutions is observed on the efficient frontier as a result of the similarities of the optimal control parameters for the different risk aversion factors. This means the operational strategy for optimising the mean, $\lambda = 0$ is similar to that required for optimising at other levels of confidences, for example, $\lambda = 2$ and so on. This does not mean that $\sigma$ is too small or that the various $\lambda$’s are near each other. On the other hand, it may also imply that the means and standard deviations of the five optimisation routines are alike, but the engineering parameters are different. Therefore, for various $\lambda$, the objective function will have numerous local optima with similar numerical values. This type of mean vs. risk outcome is preferable in robust optimisation because it eliminates the need for accepting or rejecting any level of risk since the outcomes are similar at any risk level.

In Figure 2b, the points representing the optimal cumulative oil production for the aversion factors form a convex hull known as the efficient frontier. Other points below the convex hull are non-optimal solutions encountered during the optimisation. As we move towards the downwards on the frontier, there is an increase in the level of confidence in the total amount of oil produced due to the increasing risk-aversion coefficient.

Table 1 below summarises the result from both cases. It shows that when field vertical permeability is considered as largely unknown, there is 50% confidence of producing at least 36.70MMSTB of oil. However, there is almost a 100% certainty that at least 33.68MMSTB of oil will be recovered after waterflooding if the four most significant uncertainties are taken into account during optimisation. This type of decision table presents the reservoir engineer with more realistic and valuable information for decision making and reduction of associated risks.

Table 1: Summary of results of the optimisation routines.

<table>
<thead>
<tr>
<th>Optimisation considering one uncertain variable</th>
<th>Optimisation considering four geological uncertainties</th>
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</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>$\sigma$ (MMSTB)</td>
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<tr>
<td>---------</td>
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<tr>
<td>0.0</td>
<td>0.451</td>
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<tr>
<td>1.0</td>
<td>0.452</td>
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<tr>
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<td>0.453</td>
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<tr>
<td>4.0</td>
<td>0.453</td>
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</tbody>
</table>
Figure 1 Workflow for robust optimisation of waterflood

Figure 2 Mean-variance plot for the optimisation routine involving (a) one, (b) four uncertain variable(s)

Figure 3 represents the rank of solutions from the optimisation routine based on the non-dominated sorting algorithm which took four geological uncertainties into account. Six ranks of the solutions are generated with the best solutions depicted as Rank 1. These represent the solutions that all lie on the efficient frontier and are superior to other ranks. Rank 2 solutions are superior to those in rank 3 and
so on. Generally, ranking the solutions generated by the optimisation routine allows for the differentiation of the scales of optimality of each solution, and also presents the engineer with valuable information for decision making and reduction of associated risk.

**Figure 3** Ranking of solutions for the optimisation considering four geological uncertainties.

**Conclusions**
From the results obtained, it can be concluded that the application of robust optimisation using the Markowitz classical theory leads to more realistic optimal solutions than deterministic optimisation. This is because the model incorporates reservoir control variables as well as uncertain formation rock and fluid properties. The optimisation framework enables us to obtain the control variables which result in optimum oil recovery while taking geological uncertainties into account leading to a better risk-quantified waterflood operating strategy. Using a reservoir surrogate model instead of full rigorous reservoir simulations makes the process more efficient because the computational cost is significantly reduced. Finally, to present the engineer with valuable information for decision making and reduction of associated risk, the ranking of optimisation result can be done. A non-dominated sorting algorithm was implemented to rank the solutions from the optimisation routine. This showed the different scales of optimality of each solution.

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**References**
