Introduction

Jakubowicz (1998) Internal Multiple Estimation/Elimination (IME) predicts internal multiples by convolving and correlating three primary reflections. This not only requires identifying the corresponding primaries, but also repeating this process over all (significant) multiple generating interfaces. This process might be laborious and computationally expensive and it might not be possible for interfering primary reflections (short period internal multiples). Dynamical accuracy, even assuming a perfect source wavelet deconvolution, is limited due to incorrect handling of overburden transmission effects. Replacing this repetitive process with a single closed form formula describing dynamically accurate multiples would be of great use in geophysical processing.

Recently proposed Marchenko equation based schemes calculate wavefields due to virtual sources at depth (Wapenaar et al., 2014). A multidimensional deconvolution (MDD) of said fields can remove internal multiple imprint due to the entire overburden. In order to avoid the MDD step and bring this scheme closer to Jakubowicz IME two approaches have been proposed. Firstly, van der Neut and Wapenaar (2016) showed that the MDD step of “back-projected” i.e. without actual source/receiver redatuming, wavefields can be expanded into a series. Calculating the two-way transmission correction, which requires picking an event around zero time, was the key enabler of this scheme. This approach may (in theory) work for band-limited signals provided absence of thin beds. Another draw-back of this approach is that it is not straightforward how to pick said event in practice (in particular at non-zero offsets). Secondly, Staring et al. (2018) proposed “double-focusing”, which yields a reflection response due to redatumed sources and receivers at depth, without the source- and receiver-side overburden related multiples. This scheme leaves the multiple scattering between the overburden and the target unaffected.

In this paper we derive a closed form Jakubowicz-like internal de-multiple formula, which predicts dynamically accurate internal multiples due to an entire overburden from reflection data only. Our derivation is different to the one by van der Neut and Wapenaar (2016) as it only uses fundamental scattering principles. The scheme is data driven assuming some monotonicity conditions (see e.g. Reinicke et al. (2019)) and it requires solving a set of coupled Marchenko-like equations. Our result contains Jakubowicz IME as a simplest case and bears some similarity to double-focusing if one ignores target-overburden multiple terms. The generality and minimal requirements of our approach is evidence for why many Marchenko-like methods look similar and why extensions are often rather straight-forward (provided monotonicity is observed).

Theory

In this section we first define the basic scattering principles and decompose the entire subsurface in two (shallow overburden and deeper target) parts to express the total reflection response (i.e. one already deconvolved for a source wavelet) in terms of reflection and transmission responses of the two parts. We invert this expression for the target-only reflection and show it can be expressed in terms of the solution to the Marchenko equation, and hence the total reflection response only.

We consider problems that can be written in a state space representation as $\partial X = A[X]$ (e.g. the acoustic wave equation), where $X$ is some variable (e.g. acoustic pressure and vertical velocity). As long as the linear operator $A$ is diagonalizable (e.g. with eigenstates being up-going ($U$) and down-going ($D$) acoustic waves) with some operator $L$ (Wapenaar et al., 2016), we can use the mixing term $L^{-1}\partial L$ to derive a scattering matrix $S$, which relates incident ($U$ and $D$) and scattered ($U'$ and $D'$) wavefields

$$
\begin{pmatrix}
U' \\
D'
\end{pmatrix} = \begin{pmatrix}
T^\dagger & R^\dagger \\
R^\cap & T^\cap
\end{pmatrix} \begin{pmatrix}
U \\
D
\end{pmatrix}
$$

such that

$$
\begin{pmatrix}
T^\dagger & R^\dagger \\
R^\cap & T^\cap
\end{pmatrix} \begin{pmatrix}
T^\dagger & R^\dagger \\
R^\cap & T^\cap
\end{pmatrix}^* = \begin{pmatrix}
1 & 0 \\
0 & 1
\end{pmatrix}
$$

(1)

Here $R^\dagger (R^\cap)$ and $T^\dagger (T^\cap)$, the elements of $S$, are the reflection and transmission responses from above (below) (e.g. dipole-source, monopole-receiver acoustic wave equation Green’s functions). The transmissions (and their inverses) can be written as a sum of direct and multiply-scattered (coda) parts, e.g. $T^\dagger = T^\dagger_{\text{dir}} + T^\dagger_{\text{coda}}$ and $T^\dagger_{\text{dir}} = T^\dagger_{\text{dir}} + T^\dagger_{\text{coda}}$ where $T^\dagger_{\text{dir}}^{-1} = T^\dagger_{\text{dir}}$ but $T^\dagger_{\text{coda}}^{-1} \neq T^\dagger_{\text{coda}}$. The expression to the
right in eq. (1) defines the inverse of $S$. In simple, acoustic (single mode), dissipation-free cases, the inverse is constructed out of time reversed (denoted with $^\ast$), transmission and reflection fields. In a more general setting however $^\ast$ has a wider meaning and could also denote dissipative-effectual (Wapenaar et al., 2001) or pairs of non-reciprocal media reversal (Wapenaar and Reinicke, 2019), or mode-reversal in multiple-mode supporting media (Reinicke et al., 2019). Throughout this work we use detail hiding notation (Berkhout, 1982) and we assume no free surface.

We split the subsurface into two regions shallow ($s$) and deep ($d$), with their individual reflections and transmissions (see subscripts and the left panel in Figure 1). We consider the cases where the cumulative reflection response $R$ measured above the shallow region is given by two contributions (a) the reflection due to the shallow medium $R_s$, and (b) a sequence of the transmission down through the shallow $T_s$, followed by a reflection in the deep $R_d^\prime$, a series of possible reverberations between the shallow $R_s$ and the deep $R_d^\prime$, and finally by transmission up through the shallow $T_s^\dagger$ (see the right panel in Figure 1). This is summarized by the following formula

$$R = R_s^\prime + T_s^\dagger R_d^\prime (1 + R_s^\prime R_d^\prime + R_s^\prime R_d^\prime R_s^\prime R_d^\prime + \cdots) T_s^\dagger = R_s^\prime + R_d^\prime \sum_{k=0}^{\infty} (-R_s^\prime R_d^\prime)^k \quad (2)$$

where in the latter step we have absorbed the transmissions and re-defined the reflections such that $R_s^\prime = -T_s^\dagger T_s^{\prime-1}$ and $R_d^\prime = T_d^\dagger T_d^\prime$ (note the $^\ast$ and that both $R_s^\prime$ and $R_d^\prime$ are now measured at the surface). The latter shows that the deeper reflection is always dressed with the up- and down- shallow transmission, which makes up a part of the de-multiple challenge. The other part is due to shallow-only internal multiples in $R_s^\prime$ which interfere with the later (past onset time $t_d$) $R_d^\prime$-dependent terms in equation (2).

We aim to calculate $R_d^\prime$, where as opposed to $R_s^\prime$ defined above, the shallow section does not create any internal multiples. To do that we need to define source- and receiver-side inverse transmission generators $v_s^\prime = T_s^\dagger T_{s,\text{dir}}^\dagger = 1 + \tilde{T}_{s,\text{coda}} T_{s,\text{dir}}^{\dagger}$ and $\tilde{v}_s^\prime = \tilde{T}_{s,\text{dir}}^{\dagger} T_{s,\text{dir}}^{\dagger} = 1 + T_{s,\text{dir}}^{\dagger} \tilde{T}_{s,\text{coda}}$ due to the shallow region. We solve (2) for $R_d^\prime$ and use it to get

$$\tilde{R}_d \equiv T_{s,\text{dir}}^{\dagger} R_d^\prime T_{s,\text{dir}}^{\dagger} = v_s^\prime \tilde{R}_s^\prime v_s^\prime = \tilde{v}_s^\prime (R - R_d^\prime) \left(1 - R_s^\prime (R - R_s^\prime)\right)^{-1} \tilde{v}_s^\prime \quad (3)$$

Next we show how to replace each quantity in (3), $v_s^\prime$, $\tilde{v}_s^\prime$, $R_s^\prime$ and $R_d^\prime$, by the reflection data $R$ and suitable temporal truncations. In the following we study the scattering matrix of the shallow region only. We define $v_s^\prime$ and $\tilde{v}_s^\prime$, the reflection responses of $v_s^\prime$ and $\tilde{v}_s^\prime$, and the energy/flux conservation condition

$$v_s^\prime = R_s^\prime v_s^\prime \quad \text{and} \quad \tilde{v}_s^\prime = \tilde{R}_s^\prime \tilde{v}_s^\prime \quad \text{with} \quad \tilde{v}_s^\prime v_s^\prime - v_s^\prime \tilde{v}_s^\prime = T_{s,\text{dir}}^{\dagger} \tilde{T}_{s,\text{dir}}^{\dagger} = \Sigma_s. \quad (4)$$

where $\Sigma_s$ is two-way direct transmission. The advantage here, vis-a-vis on offered in van der Neut and Wapenaar (2016), is that no event picking is needed, the formulation works well in presence of band-limitation or forward-scattering challenges. Using the definition of $v_s^\prime$ and (1) we can derive

$$R v_s^\prime = v_s^\prime + (R - R_s^\prime) v_s^\prime = v_s^\prime + U^- \quad \text{and} \quad R^* v_s^\prime = v_s^\prime - (v_s^\prime + \Sigma_s) R_s^\prime = v_s^\prime + U^+ \quad (5)$$

where we have used $U^\pm$ to draw a parallel with previous work (van der Neut and Wapenaar, 2016; Elison et al., 2019; Reinicke et al., 2019). We briefly explain the process of finding the inverse transmission generator $v_s^\prime$, further details can be found in the aforementioned publications. We assume\footnote{monotonicity always holds in acoustic 1.5D media.} monotonicity, i.e. the existence of a truncation operator $\Theta_x$, capable of separating $U^-$ and $v_s^\prime$, e.g. a temporal mute preserving the signal inside $0 < \varepsilon < t < t_d$. The small time interval $\varepsilon$ defines the extent of the temporal overlap between $v_s^\prime$ and $U^+$ in (5). Applying $\Theta_x$ to both sides we obtain a set of coupled Marchenko equations, whose solution reads

$$v_s^\prime = \sum_{j=0}^{\infty} \Omega_j [v_s^{\prime,j}] \quad \text{where} \quad \Omega_j [x] = \Theta_x [R^* \Theta_x [R \Omega_{j-1} [x]]], \quad \Omega_0 [x] = x \quad \text{and} \quad v_s^\prime = \Theta_x [R v_s^\prime] \quad (6)$$
where $v_{s,d}^+ = v_{s,d}^+ - \Theta_s [v_{s,d}^+]$ is typically assumed to be an identity operator (see the definition of $v_{s,d}^+$ above), provided the back-propagated coda $\hat{\Theta}_s coda T_{d,\text{dir}}^\dagger$ does not interfere with it. If this assumption is not valid (e.g. in the presence of SPIMs, or so-called “fast elastic multiples”) then energy conservation (i.e. observance of the right-most expression in (4)) and minimum phase conditions can be used to obtain the correct $v_{s,d}^+$ (Dukalski et al., 2019; Elison et al., 2019; Reinicke et al., 2019). Assuming source-receiver reciprocity of $R$, fields $v_{s,d}^+$ and $\tilde{v}_{s,d}^+$ are found respectively from $v_{s,d}^+$ and $v_{s,d}^-$ by interchanging of sources and receivers, if that does not hold, steps similar to the ones above can be followed for $\tilde{v}_{s,d}^+$ to derive a solution similar to (6).

After pulling $v_{s,d}^+$ through the term to its immediate left in equation (3) we get the main result of this paper

$$\hat{\Theta}_d^\dagger = \hat{\Theta}_d^\dagger (R - R_s^+) v_{s,d}^+ \left(1 - v_{s,d}^+ R_s^+ (R - R_s^+) v_{s,d}^+ \right)^{-1} = \hat{\Theta}_d^\dagger [R v_{s,d}^+] \left(1 - \mathcal{X} \right)^{-1}, \quad (7)$$

$$\mathcal{X}^- = \sum_s v_{s,d}^+= (R - R_s^+) v_{s,d}^- = (v_{s,d}^+ v_{s,d}^- - \Theta_s [v_{s,d}^+] R \Theta_s [R v_{s,d}^+])^{-1} \Theta_s [v_{s,d}^+] R \Theta_d [R v_{s,d}^+] , \quad (8)$$

where, to express everything in terms of the total reflection $R$, we used that $v_{s,d}^+ \tilde{v}_{s,d}^+ \mathcal{X}^- = \sum_s v_{s,d}^+ v_{s,d}^- \mathcal{X}^- = \Theta_d [R v_{s,d}^+]$ from equation (5), and where $\Theta_d$ is the complement of $\Theta_s$, i.e. an operator preserving $U^-$ and removing everything else.

This formulation has an easy interpretation in terms of different internal multiple contributions. The $U^- = \Theta_d [R v_{s,d}^+]$ removes the source-side ones as well as $R_s^+$ from $R$ in equation (2). Pre-multiplying with $v_{s,d}^+$ further treats the receiver side multiples (reverberations created on the way from the target), and the term $\mathcal{X}^-$ accounts for each order of multiples created by scattering between the shallow and the deep region. In the second line of eq. (7), we cast the result in the form such that the element $v_{s,d}^+ \Theta_d [R v_{s,d}^+]$ takes a form similar to the double focusing presented in Staring et al. (2018), however here without source- and receiver redatuming. The closed form of our formula allow for easy calculation of $(1 - \mathcal{X}^-)^{-1}$ using the logarithmically sped-up expansion $(1 + \mathcal{X}^-) (1 + \mathcal{X}^-^2) (1 + \mathcal{X}^-^4) (1 + \mathcal{X}^-^8) \cdots$.

Substituting (6) into (7), we get

$$\hat{\Theta}_d^\dagger \approx \Theta_d [R] + \Theta_d [R \Theta_s [R^\dagger \Theta_s [R]]] + \Theta_s [\Theta_s [R^\dagger \Theta_s [R]] \Theta_d [R] + \Theta_d [R] \Theta_s [R^\dagger] \Theta_s [R] + \Theta_d [R] \Theta_s [R^\dagger] \Theta_d [R] + \cdots \quad (9)$$

The term linear in $R$ is simply the data from the target together with all the internal multiples, the three cubic contributions can be (naively) thought of as Jakubowicz-IME-like terms. The first term coincides
with one of the terms derived by van der Neut and Wapenaar (2016) and the second term is similar to the additional one proposed by Staring et al. (2018). The third, most Jakubowicz-like looking term accounts for deep-shallow-deep internal multiples and comes from the first order expansion of $\mathcal{K}$ with $\Sigma_s = 1$.

Lastly we show how this formula reduces to Jakubowicz IME. For a single-reflector overburden, expression (6) trivializes to an identity $v_+^s = \bar{v}_+^s = 1$. Moreover, $\Theta_s[R] = R_1$ is the first primary, and $R' = \Theta_d[R]$ are all the reflection following the first event. As a result the expression (7) becomes

$$\hat{R}_d = R' (1 - \Sigma_s^{-1} R_1^* R')^{-1} \approx R' + \alpha_1 R'R_1^* R' + \alpha_2 R'R_1^* R'R_1^* R' + \cdots$$

(10)

where $\Sigma_s = 1 - R_1^* R_1$, which is a Jakubowicz IME like prediction of all the orders of multiples which share a downward reflection at a first reflector and $\alpha_1$ and $\alpha_2$ denote amplitude matching filters.

Conclusion

We have derived a general de-multiple series expansion formula, which bridges our understanding of Jakubowicz IME-type and Marchenko equation-based approaches. In our derivation we have only used the scattering principles and a monotonicity assumptions. This formula is hopefully more intuitive and more general than the previous proposals. The precise relation of our formula, which handles all (order) overburden internal multiples, to inverse scattering series ‘eliminator’ (Weglein, 1997; Zou et al., 2019) remains to be investigated.

References


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