Introduction

Effects of crack properties (e.g., crack density and crack aspect ratio) on the elastic properties of rocks have always been the focus of petrophysics and seismic exploration research due to the facts that cracks are widely distributed in all kinds of rocks, and have significant influence on the elasticity and flow properties of reservoir rocks. To predict and evaluate the fractured reservoirs accurately, petrophysical models are compulsory to relate the physical properties of rocks to the crack properties.

Although much research have been done on the elastic properties of fractured rocks, most of them only consider the simplified cases of isotropic background and aligned cracks, and the anisotropic elastic properties of rocks with transversely isotropic (TI) background permeated by 3-Dimensional (3D) inclined cracks (i.e., the cracks not only intersect with the background with a dip angle between the cracks and the isotropic plane but also with a rotation angle between cracks and plane normal to the isotropic plane) and the randomly orienting cracks are still yet to be studied.

This work aims to study the effects of 3D inclined and randomly orienting penny-shaped cracks on the elastic anisotropy of rocks with TI background. To achieve this goal, we first derive equations for the excess compliance matrix of the 3D inclined cracks and then the elastic properties of rocks with TI background permeated by randomly orienting cracks are obtained through the non-interaction approximation. We further study the effects of the two inclined angles on the elastic properties of fractured rocks with TI background, and the influences of crack density and aspect ratio are finally investigated.

Theory

To calculate the elastic properties of rocks with TI background permeated by dry inclined penny-shaped cracks, the excess compliance tensor for the penny-shaped cracks can be expressed in the following form (e.g., Kachanov, 1992):

$$Z_{ijkl} = \frac{1}{4} (n_i B_{jk} n_k + n_j B_{ik} n_k + n_j B_{ik} n_k + n_i B_{jk} n_k),$$

where $Z_{ijkl}$ is the excess compliance tensor of cracks, $B_{ij}$ is the second rank crack opening displacement (COD) tensor (Guo et al., 2019), $n_i$ is the unit crack normal, which can be expressed as follows for the 3D inclined penny-shaped cracks,

$$n = (-\sin \theta \sin \varphi, -\sin \theta \cos \varphi, \cos \theta),$$

where $\theta$ is the dip angle between cracks and the isotropic planes, and $\varphi$ is the rotation angle along with the symmetry axis.

Since we have got the elastic properties of single crack, the excess compliance matrix of dry multiple 3D orienting cracks can be obtained based on the non-interaction approximation (Sevostianov et al., 2005),

$$Z_{ijkl}^{\text{NI}} = \sum Z_{ijkl}(\theta, \varphi),$$

where $Z_{ijkl}^{\text{NI}}$ is the sum of the excess compliance tensor for each dry crack as functions of dip angle $\theta$ and rotation angle $\varphi$ with $Z_{ijkl}$ given in Equation (1).

Then, according to the linear-slip theory (Schoenberg et al., 1995), the effective elastic properties for dry TI rocks permeated by randomly orienting cracks can be given as

$$M_{ijkl}^{\text{eff}} = M_{ijkl} + Z_{ijkl}^{\text{NI}},$$

$$C_{ijkl}^{\text{eff}} = (M_{ijkl}^{\text{eff}})^{-1},$$

where $M_{ijkl}$ is the compliance tensor of dry TI background of fractured rocks, $M_{ijkl}^{\text{eff}}$ and $C_{ijkl}^{\text{eff}}$ are the compliance matrix and the stiffness matrix of the dry TI fractured rocks with randomly orienting cracks, respectively. The effective stiffness matrix of the dry TI fractured rocks with 3D inclined cracks can be obtained employing the Equations (4) and (5) by substituting $Z_{ijkl}^{\text{NI}}$ to $Z_{ijkl}(\theta, \varphi)$. 
Specially, the elastic properties of saturated cracks can be obtained using the Brown-Korringa equation (Brown and Korringa, 1975) based on the compliance matrix of the dry cracks:

$$Z_{ij}^{\text{sat}} = Z_{ij} - \frac{\sum_{m=1}^{3} Z_{im} \sum_{n=1}^{3} Z_{jn}}{\sum_{m=1}^{3} \sum_{n=1}^{3} Z_{mn} + \phi (1/K_f - 1/K_s)} , \quad i, j = 1, 2, 3...6, \quad (6)$$

where $K_f$ is the bulk modulus of the pore fluid, and $K_s$ is the generalized bulk modulus of the TI background which can be calculated through the following equation:

$$K_s = (C_1^2 + C_{12} C_{33} - 2C_{13}^2) / (C_{11} + C_{12} + 2C_{33} - 4C_{13}), \quad (7)$$

where $C_{ij}$ are the components of the elastic matrix of the TI background.

Having got the elastic properties of saturated cracks, the elastic properties for saturated fractured rocks with TI background permeated by randomly orienting cracks or 3D inclined cracks can be obtained using the above equations.

**Simulation results**

Having developed theoretical models for the elastic properties of rocks with TI background permeated by 3D aligned and randomly orienting penny-shaped cracks, we present numerical examples in this section to illustrate the effects of 3D inclined penny-shaped cracks and 3D randomly orienting inclined cracks on the anisotropic elastic properties of fractured rocks.

Firstly, the effects of rotation angle $\phi$ are illustrated by calculating the elastic matrix through changing $\phi$ between $0^\circ$ and $360^\circ$ under a given dip angle $\theta$ as shown in Figure 1. The results indicate that the rotation angle $\phi$ have significant impacts on the elastic anisotropy of rock with 3D inclined cracks, and therefore should be considered when studying the elastic properties of fractured rocks with TI background permeated by 3D inclined penny-shaped cracks.

**Figure 1** Variations of the elastic coefficients of the fractured rock with TI background permeated by dry and saturated inclined cracks with rotation angle $\phi$ under different dip angle $\theta$. (a) and (d): $\theta = 0^\circ$, (b) and (e): $\theta = 45^\circ$, and (c) and (f): $\theta = 90^\circ$. 

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Figure 2 Variations of velocities for the fractured rock with TI background permeated by dry penny-shaped cracks with incident angle. (a): $P$-wave velocities in the $x$-$z$ plane, (b): $SH$-wave velocities in the $x$-$z$ plane, (c): $SV$-wave velocities in the $x$-$z$ plane, (d): $P$-wave velocities in the $y$-$z$ plane, (e): $SH$-wave velocities in the $y$-$z$ plane, (f): $SV$-wave velocities in the $y$-$z$ plane, (g): $P$-wave velocities in the $x$-$y$ plane, (h): $SH$-wave velocities in the $x$-$y$ plane, (i): $SV$-wave velocities in the $x$-$y$ plane.

Then, Figure 2 shows the variations of the anisotropic velocities for the rock with TI background permeated by dry randomly orienting cracks with the incident angles (i.e., $\gamma$, $\delta$ and $\beta$, respectively, where $\gamma$ and $\delta$ are the angles between the incident direction and $z$-axis in the $x$-$z$ plane and $y$-$z$ plane, respectively, and $\beta$ is the angle between the incident direction and $x$-axis in the $x$-$y$ plane.). From Figure 7, we can see that the increasing crack density reduces the velocities of the fractured rock with dry randomly orienting cracks, and velocities in the $x$-$z$ plane and $y$-$z$ plane are more sensitive to the variation of crack density, which can be explained through the changes in the effective stiffness coefficients caused by the variations of crack density.

Figure 3 Variations of $P$-wave velocities for rocks with saturated cracks in the $x$-$z$ plane with (a) crack density and (b) aspect ratio, respectively.

The above analyses have proved that the crack properties have great influences on the elastic properties and velocities of the fractured rocks with randomly orienting cracks. Hence, more detailed
researches are carried out to give the relations between the velocities and the crack parameters of fractured rocks with randomly orienting cracks for inverting the crack properties (i.e., crack density and aspect ratio).

Figure 3 shows the relationships between the crack properties and the P-wave velocities with different incident angles for rocks with saturated cracks (i.e., the red points represent the P-wave velocities when $\gamma$ equals 0°, and the blue points represent the P-wave velocities when $\gamma$ equals 90°). The velocities given in this figure have good linear relationships (with squared correlation coefficients $R^2$ greater than 0.92) with crack density and the aspect ratio, which are expressed as follows:

\begin{align*}
  V_{p90} &= -1.85 \epsilon + 4.34, R^2 = 0.995, \\
  V_{p90} &= -1.9 \epsilon + 3.68, R^2 = 0.998, \\
  V_{p90} &= -2.02 \alpha + 4.26, R^2 = 0.926, \\
  V_{p90} &= -3.77 \alpha + 3.62, R^2 = 0.98, 
\end{align*}

where $V_{p90}$ and $V_{p0}$ represent velocities of the P-wave that are parallel and vertical to the z-axis in the x-z plane, respectively. Therefore, based on the velocities from the acoustic logging, the crack density and aspect ratio can be calculated using the Equations (8) to (11).

Conclusions

In this work, we study the elastic properties of rocks with TI background permeated by 3D inclined cracks and randomly orienting cracks. Based on the solutions for the effective elastic properties of rocks permeated by aligned cracks, we derive the new compliance matrix for dry and saturated 3D inclined cracks and present theoretical models for the elastic properties of rocks with TI background permeated by dry and saturated 3D randomly orienting cracks.

From the derived models, we analyze the effects of $\phi$ on the elastic properties of fractured rocks with dry and saturated cracks. The results show that, rotation angle $\phi$ have significant impacts on the elastic anisotropy of rock with 3D inclined cracks, and therefore should be considered when studying the elastic properties of fractured rocks with TI background permeated by 3D inclined penny-shaped cracks. To illustrate the effects of randomly orienting cracks on the elastic properties of fractured rocks, we demonstrate the variations of the elastic coefficients with crack density for rocks permeated by dry and saturated randomly orienting cracks. The results show that both the elastic coefficients and velocities decrease with the increasing crack density.

Based on the effects of crack density and aspect ratio on the velocities of fractured rocks, we obtain the relationships between the crack properties and P-wave velocities in different directions from acoustic logging. The models proposed in this paper are convenient to use, and can be applied in the seismic inversion scheme for the characterization of the fractured formations.

References