Introduction

Successful utilization of petroleum resources requires a comprehensive monitoring of the subsurface, where time-lapse (4D) seismic is among the most important sources of knowledge. However, this requires that the 4D seismic attributes such as time- and impedance-shifts are adequately interpreted, for a proper quantification of the geomechanical changes caused by the depletion. In the early stages of the millennium 4D seismic was proven not only to be influenced by the reservoir itself via the depletion process, but also by the geomechanical changes in the non-reservoir rocks. The overburden in particular is significantly influenced, which is associated with subsidence, stress changes and potential fault reactivation manifesting as detectable time-shifts (e.g. Hall et al., 2002; Kenter et al., 2004; Zoback and Zinke, 2002). The main goal with the processing of the 4D seismic data is a proper separation of the velocity changes and the strains, where the offset dependence of time-shifts turns out to be essential (Landrø and Janssen, 2002). This requires a physical rock model that links geomechanical changes to time-shifts. In this respect the R-factor model has been widely used (Hatchell and Bourne, 2005; Røste et al., 2005). This model assumes that the normalized vertical velocity change (ΔVv) is linearly related to the vertical strain (εz; defined positive for compaction):

\[ R_v = \frac{\Delta V_v}{V_v \epsilon_z} \]  

(1)

Most commonly the \( R_v \) is used to interpret post-stack data, even though this model has also been used on pre-stack data (MacBeth et al., 2018). Although the \( R \)-factor has been regarded as an "earth constant" in some studies, most publications now regard the \( R \) as a spatially varying quantity for which specific values are linked to specific layers or formations (e.g. De Gennaro et al., 2008). The increased focus on the offset-dependence of time-shifts has also challenged the original \( R \)-factor model as it lacks a tie to non-vertical strains.

A rock physics model to be used to invert 4D seismic data, should have a physical validation that enables reliable generalization and predictability of results. In this work we discuss the \( R \)-factor and some generalizations of this model by considering experimental data obtained in laboratory experiments on overburden field shales. We also demonstrate how a higher order (non-linear) optimized elastic dynamic stiffness tensor may adequately describe the strain dependence of velocities, which provide a good fit to the offset-dependence of time-shifts from a North-Sea shale.

Experimental method

All measurements and further analysis assume TI symmetry where the symmetry axis is normal to the horizontal (bedding) plane. The ultrasonic data were acquired on single plugs, where the P-wave velocities were measured along multiple ray (group) angles with different stress variations (stress paths) around the in-situ stress state: constant mean stress change (CMS), vertical stress change (triaxial); zero radial strain \( K_0 \) and isotropic stress change (cf. Figure 1). The offset angle refers to the vertical axis, i.e. 0° is vertically and 90° is in the horizontal plane. During the stress cycles the pore pressure was undrained, which is assumed to be most representative of low permeability overburden rocks (cap). The field shale cores were stored as "seal-peels" at ambient conditions prior to testing (Bakk et al., 2019).

Figure 1 In-situ stress path variations (schematic). Prior to each undrained stress cycle the samples were drained back to the in-situ pore pressure (the reference state). The vertical stress was changed by 5 MPa in all cycles.
Discussion

The assumption of a constant \( R \)-factor model is appealing since it explicitly relates the velocity change and the strain. This implies that only the vertical strain is a variable when the time-shifts are considered. However, it turns out that the \( R \) is not constant throughout the subsurface. Hatchell and Bourne (2005) attributed different \( R \)-factors to the depleting reservoir and to the surroundings under extension. It should be noted that the depleting reservoir undergoes significant and potentially inelastic compaction in addition to fluid substitution, which makes the comparison to its surroundings difficult. De Gennaro et al. (2008) show that even within the overburden \( R \) is significantly varying in the Elgin and Franklin Fields. The overburden in these fields exhibit large mechanical contrasts between the layers, and between the overburden and the reservoir. Such heterogeneities in the static stiffness throughout the subsurface may also lead to complex stress variations due to the reservoir depletion, i.e. the change in magnitudes (Mulders, 2003) and directions (Herwanger et al., 2007) of the principal stresses. Laboratory experiments on field cores may be used to systematically study the impact of different stress paths in the overburden. In Figure 2(a) we show the significance of stress path dependence on the \( R \)-factor. The data in this case are fitted by using Hooke’s law to relate the stress and strain sensitivities of velocities (Holt et al., 2018). In a field case one will experience a range of different stress paths depending on the subsurface location of interest and the specific field (layer properties and production history). Stress and strain data may be obtained from geomechanical simulations coupled to seismic analysis. One should note that often only isotropic stress variations are investigated in laboratory tests, for which the \( R \)-factors may lead to misinterpretation of the 4D seismic data. An isotropic stress path implies very different \( R \)-factors as compared to, for example, uniaxial stress (\( \kappa = 1 \) versus \( \kappa = 0 \) in Figure 2(a)).

\[ \Delta t = -\varepsilon_z \cos^2 \phi \Delta V(\phi) V(\phi), \]

which is valid for anisotropic velocities (\( V(\phi) \)). By inserting the \( R_z \) from Eq. (1) into Eq. (2) this simplifies to:

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**Figure 2** Laboratory data from overburden field shales. (a): The average values of \( R_z \) from three different shales, calculated according to Eq. (1) (based on the vertical strain), as function of stress path \( \kappa \) (ratio between the horizontal and vertical stress changes). The fit is obtained by converting the linearized stress sensitivities to \( R \)-factors by optimizing the Poisson’s number (full line). (b): \( R \) for different stress paths as function of ray angle (full lines) for a North-Sea shale. The \( R_\phi \) (open symbols and broken connection lines) are based on the path strain (replacing \( \varepsilon_z \) in Eq. (1)).

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The huge potential of pre-stack offset data has in practice challenged the \( R \)-factor as it is not obvious why the vertical strain is unique. Kudarova et al. (2016) discussed how to generalize the \( R \)-factor to account for the path strain (\( R_\phi \)), defined as the normal strain at ray angle \( \phi \), as an alternative to the traditional vertical strain dependence (\( R_z \)). They analysed the offset-dependence of time-shift data obtained from the Mars and the Shearwater Fields. In Figure 2(b) we compare these two approaches on ultrasonic laboratory data of a North-Sea overburden shale. The \( R_\phi \) has a strongly fluctuating trend as function of offset. For the \( K_0 \) stress path, \( R_\phi \) is rapidly increasing versus offset as the path strain becomes zero for 90°. On the other hand, the \( R_z \) is independent of ray angle for a given stress path according to Eq. (1). If we consider a single homogeneous isotropic layer and straight ray paths, the relative time-shift (equal to time-shift here) is obtained by a differentiation of the (pre-stack) two-way travel-time:
where the \( R_z \) implies an isotropic strain sensitivity. By using the same dataset as in Figure 2(b), we calculate the experimental relative time-shifts according to Eq. (2) (cf. Figure 3(a)). The experimental data exhibit qualitatively different trends for the respective stress paths, where the CMS stress path exhibits the largest offset gradient. In the same plot we fix the \( R_z \) for each stress path to calculate the time-shifts cf. Eq. (3). Consequently, the time shifts for zero offset are perfectly matched, but the offset trends do not match the experimental data neither qualitatively nor quantitatively, as expected (Figure 3(a)). Thus, a model capturing strain dependence must also include offset dependence of strain sensitivity, in addition to the stress-path dependence. One should note that by using anisotropic offset velocity changes in the \( R_z \), i.e. replacing \( V_z \) with the angular velocity in Eq. (1) similar to Hawkins et al. (2008), the time-shift trends are equally variable and do not adequately match the experimental data.

Figure 3 Relative time-shifts for vertical unloading versus ray angle for different stress paths (cf. Figure 1) with linear trendlines. The reference data are calculated from ultrasonic measurements on a North-Sea overburden shale cf. Eq. (2) (filled symbols; full trendlines). (a) Time-shifts with the experimental value of \( R_z \) for each stress path cf. Eq. (3) (open symbols; broken trendlines). (b) Time-shifts based upon an optimized fit to the TOE model of Fuck and Tsvankin (2009) (open symbols; broken trendlines).

To better match the time-shift data, third order elasticity (TOE) models can be used. The model of Prioul et al. (2004) is appealing with only three additional elastic constants. This model does not match the time-shifts obtained from laboratory shale tests, as its underlying assumption of isotropic strain sensitivity is an oversimplification. However, the TOE of Fuck and Tsvankin (2009) with VTI symmetry of the strain dependence gives a good fit to the experimental time-shift data as shown in Figure 3(b). The latter model involves effectively eight independent TOE parameters (seven for the P-waves) to describe VTI anisotropic strain sensitivity of the velocities. By including more experimental data one may find correlations between the parameters to make such model better suited for 4D seismic inversion.

Conclusion

Quantification of the strain sensitivities of velocities is required to adequately invert pre-stack 4D seismic data for static strains in and around a depleting reservoir. Laboratory tests on field shale cores are conducted where different stress paths are systematically varied. The widely used \( R \)-factor model, assumed to represent the strain sensitivity of P-wave velocities, is shown to be significantly influenced by the stress path. Shales are inherently anisotropic in terms of both static and dynamic stiffnesses, and we also find evidence that shales exhibit anisotropic strain sensitivities. We demonstrate a satisfying fit to offset velocity data from laboratory shale tests when we include VTI symmetry in a third order elastic dynamic stiffness model, contrary to the \( R \)-factor model. Further data from such experiments together with an adequate constitutive description of the strain sensitivity, may give a good basis to better understand these systems and how this may be used to improve 4D seismic field data analysis.

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