**Introduction**

Over recent years, full waveform inversion (FWI) has become an important tool for estimating high-resolution model parameters that faithfully describe the properties of the subsurface. Since its introduction in the early 80’s by Lailly (1984) and Tarantola (1984), many authors have introduced multiple techniques to implement waveform inversion through various approximations.

In the context of exploration seismology and more specifically in velocity model building our main goal is to calculate the parameters that describe the physical properties of the subsurface that govern elastic wave propagation and explain the observed seismic field data. The parameters referred to as the Earth model, are subsequently used to produce a realistic representation of the subsurface ‘image’ through the process of migration.

Almost all FWI approaches use some sort of source wavelet information to extrapolate the down-going wavefield: this source wavelet could either be modelled, extracted from the recorded data during the pre-processing, or in some cases, a zero-phase band-limited synthetic wavelet can be used instead. Alternatively, the source information can be regarded as an unknown of the inverse problem (Pratt, 1999; Virieux and Operto, 2009) and be updated within the FWI procedure itself (Sun et al., 2014; Rickett, 2013; Wang et al., 2017).

The importance of reliable source wavelet information during the FWI implementation has been widely implied or briefly mentioned by multiple authors e.g. (Virieux and Operto, 2009; Jones, 2018), but little detailed study of the effects of poor wavelets has been presented in the literature.

After presenting a brief description of where FWI uses the source wavelet, we will describe some of the mechanisms leading to source wavelet estimation error. The current literature suggests that the source signature plays a crucial role in the inversion, but how crucial this role is, is a matter of question. In this work we quantitatively assess the effect of source wavelet error on both the resulting velocity field and on the depth of horizons in the associated migrated images.

**Method**

FWI is a seismic parameter estimation technique that generates, or searches for, a high-fidelity high-resolution Earth model that explains the kinematics of the recorded data, and ideally utilizes the full waveform, which means that the travel-time, phase or amplitude information is involved in the inversion process. Waveform inversion simulates the wavefield by attempting to solve the wave equation: in practice this is usually an acoustic approximation of the wave equation (i.e. density change is ignored and shear mode conversion and propagation are not accounted for). The acoustic FWI approach that inverts only for the velocity model is currently the commercial standard (Virieux and Operto, 2009; Warner and Guasch, 2016; Jones, 2018). The high-resolution P-wave velocity model retrieved by FWI is then typically used for subsequent preSDM.

Given that changes in the source wavelet will alter the resulting inverted velocity, we want to assess the nature and degree of these changes. Here we assess the sensitivity to source wavelet error by considering two norms. Firstly, the mean residual velocity error $C_v(m)$, being the average absolute difference between some reference velocity model and the model obtained by FWI using the perturbed source wavelet:

$$C_v(m) = \frac{1}{\sum_{\text{inlines}} \sum_{\text{crosslines}} \sum_i} \left| v_{\text{ref}} - v_i \right|$$

And secondly via assessing the vertical depth positioning error $C_z(m)$, for some reference horizon that would result from using the perturbed rather than the reference velocity model:

$$C_z(m) = \frac{1}{\sum_{\text{inlines}} \sum_{\text{crosslines}}} \left| z_{\text{ref}} - z_i \right|$$

The analysis is conducted initially for a synthetic data set and then for a field data case study example from the Norwegian Sea (Singh et al., 2020). In the synthetic trials, we independently perturb wavelet phase and then wavelet time-shift, and repeat this exercise initially for the exact reference model, and then from using a smoothed version of the reference model. For the field data, where the ‘true’ model
and source wavelet are unknown, we make do with a best estimate of both, and then derive a series of different, but reasonable, source wavelets to again assess the changes in the velocity model resulting from FWI using these wavelets. In an industrial FWI project, we’d process the entire 3D data volume though typically 50 – 100 iterations of FWI, using different variants of FWI (travel-time norm, data difference norm, refraction, reflection), and employing staging strategies for differing offset and frequency ranges. The forward modelling would be conducted using parameters for velocity, anisotropy, and perhaps density and Q. Such a project would take a few months, consuming a large amount of CPU-time, and at best employ a source wavelet that was updated occasionally as the model converged. However, due to the practicalities of time and CPU constraints for this study, here we have limited the procedure for the 3D field data as follows:

1) Taking the final model from a successful commercial FWI project (Singh et al., 2020), modify it by inverting a swath of data to a broader bandwidth using the final ‘production’ source wavelet. The inversion scheme used is that of a conventional data-difference least-squares norm, employing the refracted (transmitted) wavefield. Hence the data input to the inversion are restricted to a window of data that primarily captures the diving wave energy.

2) Re-run the inversion with a suite of alternative source wavelets, and allow the FWI to run for a maximum of ten iterations for each alternative source wavelet. Note that we are actually only fine-tuning an already good model, rather than starting from scratch with each new trial source wavelet.

3) Compute metrics comparing the inverted velocities obtained for the reference and trial source wavelets, by assessing the average absolute velocity differences in the region penetrated by the refracted wavefield.

4) Using each of the derived velocity models, perform 3D preSDM and compare the depths for target horizons, between the reference and each of the trial results. This depth error analysis is conducted in the vicinity of a well location, providing a ‘ground-truth’ reference for both depth and interval velocity.

Results

The synthetic data were taken from the non-salt portion of the well-known BP-2004 AIT model (Billette and Brandsberg-Dahl, 2005). This is used as the reference for assessing mean absolute velocity change and residual absolute depth error, when performing FWI with a perturbed source wavelet. Figure 1 displays a zoomed portion of a shot gather taken from above a ‘gas’ anomaly showing the ‘field data’ with an overlay of data modelled with correct zero-phase wavelet, and also with an overlay of data modelled with erroneous 45° phase rotated wavelet. In Figure 2 are shown the velocity errors (from equation 1) versus phase shift and also time shift for a suite of perturbed source wavelets. Also shown is the depth position along the seismic line for a reference horizon at 2km depth, from the image associated with the reference wavelet and also for the ±30° and ±45° phase shifted source wavelets. For this synthetic case, the depth error is acceptably small (about 14m: within typical image depth uncertainty range: Vlassopoulou et al. 2019) for the ±30° phase shift.

Figure 1. Zoom of portion of a shot for field data with overlay of data modelled with correct zero-phase wavelet (left), and right, overlay of data modelled with erroneous 45° phase rotated wavelet.
Figure 2. Left: plot of velocity errors (from equation 1) versus phase shift (blue curve) and also time shift (red curve). The wavelet is shown: not the same scale as the time-shift axis. Right: depth position of the 2km-deep reference horizon along the seismic line for the reference, ±30° and ±45° phase shifted wavelets. The depth error is acceptably small (within typical image depth uncertainty range) for the ±30° phase shift.

For the field data example (relatively flat-lying sediments over dipping unconformable beds) FWI revealed a near-seabed low velocity layer, along with other geologically conformable updates (Singh et al., 2020). The original source wavelet used in the production FWI project (derived from a deghosted near trace stack at the sea bed) was perturbed to create several other ‘reasonable’ source wavelets. Only the refracted wavefield is used in the inversion: the offset range being limited to 2 – 5.5 km for the early arrivals. The analysis of errors was conducted for events down to the maximum depth of penetration of the diving-wave energy – about 1600 m (as we only considered refraction FWI in this study). Firstly, we analyse the errors in a 4kmx4km patch centred on the well position (for two representative horizons), and then for the width of the entire section for a deeper horizontal marker. Figure 3 displays the average velocity ‘error’ (change) with respect to the production ‘best’ FWI model (blue curve), and the average vertical depth error corresponding to the differing wavelets, measured for the reference horizon at about 1300 m depth in the vicinity of the well (green curve). The corresponding source wavelets are also shown. Wavelets #1 - #4 are produced by tapering the reference wavelet differently; #5 is a 7 Hz Ricker wavelet with 240° phase rotation; #6 is derived using a deconvolutional procedure. The worst results (wavelets #7 and #8) are obtained using a 7 Hz Ricker wavelet with 130° and 0° phase rotations, respectively: these waveforms least resemble the reference wavelet. The last three results are obtained using the data-derived reference wavelet after 30°, 50° and 70° phase rotations for comparison to the synthetic tests.

Conclusions
To a certain extent, the velocity and associated image depth perturbations resulting from slight wavelet change can be considered as acceptable, as compared to the depth errors associated with tomographic uncertainty. It is only when we exceed perhaps a ±30° source phase error or a very poor source wavelet estimate, that we suffer unacceptable changes in the result. In other words, if the location of the wavelet peak energy in the modelling is similar to that of the field data, then FWI is relatively robust with respect to source error. However, for more complex geological environments with rapid lateral velocity variation and steeply dipping events (e.g. salt), the analysis presented here would need to be repeated in order to draw representative conclusions.

Acknowledgements
We would like to thank Jeet Singh, Victoria Valler, Stuart Greenwood, Chao Wang, and Rodrigo Felicio, for their help with this work, to John Brittan, Jacques Leveille, and Carlos Calderon for proof reading, and to OMV and partners for the data example, and ION for permission to present the work.
Figure 3. Average absolute velocity ‘error’ (change) with respect to the ‘best’ FWI model (blue curve) and associated average vertical depth error measured near the well at depth 1300 m (green curve). Bottom: display of the original estimated wavelet (1-2-20-30 Hz) and the various source wavelets (1-2-7-9 Hz) used in the study (note: the display time-alignment of the wavelets is unimportant).

References


