Introduction

Precise numerical modelling of the wave propagation in geological media is crucially important for seismic inversion (Li and Alkhalifah (2020)) and migration (Zhou et al. (2019)) techniques. Due to the sedimentation procedure the upper part of geological sections is anisotropic. For this reason, the velocity of the wave propagation depends on the propagation direction. One of the major cases is the vertical transverse isotropic (VTI) media. Different approaches were proposed for precise simulation of wavefields in such media (Wang et al. (2018); Liu et al. (2018)).

In this work we propose a novel approach for simulation of the dynamic behavior of the VTI fractured medium. It relies on the grid-characteristic numerical method on rectangular grids. It continues our activity of extending this method for different types of rheology (Golubev et al. (2021a); Golubev et al. (2020a); Golubev et al. (2021b); Beklemysheva et al. (2021)). The method was successfully implemented as a part of our in-house software RECT and applied to the improved Marmousi2 model.

Method and Theory

Vertical Transverse Isotropic Medium

The second law of the Newtonian mechanics for an infinitesimal volume of medium can be written as follows:

\[ \rho \ddot{\vec{v}} = \vec{\nabla} \cdot \sigma + \vec{F}. \] (1)

Here \( \rho \) is the medium density, \( \vec{v} \) is the velocity vector, \( \sigma \) – the stress tensor, \( \vec{F} \) is the external force. Tensor \( \sigma \) is symmetric and consists of the following components in the two-dimensional case:

\[ \sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix}. \] (2)

For the strain tensor \( \varepsilon \) (with components \( \varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{xy}, \varepsilon_{yx} \)) the following relation holds:

\[ \varepsilon_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), \quad i, j \in \{x, y\}. \] (3)

Here \( \vec{u} \) is the medium displacement and \( \vec{u} = \vec{v} \).

In the isotropic case, the stress-strain relation is described with two independent Lame parameters \( \lambda, \mu \) according to the formula

\[ \sigma_{ij} = \lambda \delta_{ij} \varepsilon_{ii} + 2 \mu \varepsilon_{ij}, \] (4)

where \( \delta_{ij} = 1 \) if \( i = j \) else 0. (5)

For the two-dimensional VTI medium the stress-strain relation is more complicated and can be written as follows (Carcione et al. (1988)):

\[ \begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{pmatrix} = \begin{pmatrix} C_{11} & C_{13} & 0 \\ C_{13} & C_{33} & 0 \\ 0 & 0 & C_{44} \end{pmatrix} \begin{pmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{pmatrix} \] (6)

Here \( C_{11}, C_{13}, C_{33}, C_{44} \) are the material parameters.

Numerical Method

In this work, the grid-characteristic numerical method on structured rectangular grids was used. Its detailed description for isotropic media can be found in the work (Favorskaya et al. (2018)). Here we have extended this approach for the VTI media. Below is the short summary of it.

Firstly, we rewrite the system in the canonical form: \( \ddot{\vec{q}} + A_1 \dot{\vec{q}} + A_2 \vec{q} = \vec{f} \), where \( \vec{q} \) denotes the vector of the unknown functions, \( A_1 \) and \( A_2 \) are matrices defined by medium parameters, \( \vec{f} \) is defined by the external force. Then we utilize a coordinate splitting technique:
\[ \hat{\sigma}_i + A \hat{\sigma}_x = 0 \text{ and find } \hat{\sigma}^{n+1/2} \]

2. Solve \[ \hat{q}_i + A_2 \hat{q}_y = 0 \text{ and find } \hat{q}^{n+1} \]

3. Solve \[ \hat{q}_i = \tilde{f} \text{ and find } \hat{q}^{n+1} \]

The original system of PDEs is hyperbolic, so each matrix \( A_1, A_2 \) has a full set of eigenvectors, and the spectral decomposition may be applied: \( A_i = \Omega_i^{-1} \Lambda_i \Omega_i \), where lines of the matrix \( \Omega_i \) are left eigenvectors of the matrix \( A_i \), and the matrix \( \Lambda_i^{-1} \) is its inverse. The diagonal matrices \( \Lambda_i \) consist of eigenvalues 0, \( \pm \sqrt{C_{11}/\rho}, \pm \sqrt{C_{33}/\rho}, \pm \sqrt{C_{44}/\rho} \), the absolute values of which are the medium velocities.

We can denote Riemann invariants \( \tilde{\omega} = \Omega \hat{q} \), solve a number of independent transport equations \( \tilde{\rho} + \Lambda \tilde{\rho} = 0 \) (\( \xi \in \{x,y\} \)), after which return to the initial unknowns \( \hat{q} = \Omega^{-1} \tilde{\omega} \). Thus, knowing initial distribution and the necessary boundary conditions, we obtain a direct time-stepping algorithm. Time step \( dt \) must satisfy CFL condition \( dt < h/\lambda_{\text{max}} \), where \( h \) is grid step.

**Fractures**

We consider the model of infinitely narrow fractures. The following conditions must be satisfied on the two sides of the crack:

\[
\begin{align*}
& (\tilde{f}_{ab}, \tilde{n}) = -(\tilde{f}_{ba}, \tilde{n}), \\
& (\tilde{f}_{ab}, \tilde{\tau}) = 0, \\
& (\tilde{f}_{ba}, \tilde{\tau}) = 0, \\
& (\tilde{v}_a, \tilde{n}) = (\tilde{v}_b, \tilde{n}).
\end{align*}
\]

Here indices \( a \) and \( b \) denote the two sides of the fracture; unit vector \( \tilde{n} \) is normal to the crack, unit vector \( \tilde{\tau} \) is tangential to the crack, \( (\tilde{n}, \tilde{\tau}) = 0 \); vector \( \tilde{f} \) defines unit force: \( \tilde{f}_{ab} = \sigma_a \cdot \tilde{n}, \tilde{f}_{ba} = \sigma_b \cdot (-\tilde{n}) \).

The formulae above can be considered as the interface conditions for an arbitrary geometry. A possible implementation of such interface for curvilinear grids using a boundary corrector is described, for example, in the paper by Golubev et al. (2020c). However, most of geological fractures are vertical (or subvertical) due to the massif stress state. That is why in this work only the case of purely vertical fractures was considered. The generalization for arbitrarily oriented cracks exists for the isotropic case, and its description along with the accuracy analysis has been done by Khokhlov and Stognii (2020). It should be noted that the continuum approach may be used (Golubev et al. 2020b).

In order to take the crack into account in our calculations, we split each mesh node located on the fracture artificially into two nodes: 'a' and 'b', corresponding to each side of the fracture. The numerical scheme described in the previous subsection iterates over the computational domain and treats these duplicated nodes separately. Then a special correction is performed, which imposes the appropriate interface conditions (7).

**Examples**

We have implemented the technique described above and used it to simulate the well-known Mar-moussi 2 two-dimensional model with a cluster of cracks. Thomsen parameters (Thomsen 1986) of the anisotropy were calculated by the empirical relations (Yan and Sava 2009): \( \varepsilon = 0.25 \rho - 0.3 \) and \( \delta = 0.125 \rho - 0.1 \). Here density is measured in \( g/cm^3 \).

It was covered by the square grid with 3400 x 700 nodes with spatial distance equal to 5 m. At the top boundary the free surface conditions were explicitly set. The source was buried by 5 m in the center of the model and had the Ricker profile with the major frequency 30 Hz. Eleven vertical fluid-filled fractures with 500 m length were placed inside the model to cover the rectangle area [4000 m, 4500 m] at the depth of [1000 m, 1500 m].
Results

For clarity, in all figures the spatial distribution of the density in the Marmousi2 model was used as a background. In the Figure 1 the absolute value of the normalized velocity is represented. It was calculated as the ratio of velocity to its maximum over the whole domain. The wavefield is very complex: surface and body waves (incidental, reflected from all layers and diffracted) are visualized. It is impossible to identify the response from fractured object on it. In the Figure 2 the normalized signal difference is presented. It was calculated as the absolute value of difference between signals registered for fractured anisotropic and non-fractured anisotropic media. It was normalized according to the same procedure. In both figures the position of the fractured region is highlighted with the white rectangle.

**Figure 1** The density of the Marmousi2 model (in color) and the velocity modulus (in gray scale) for the final time moment. The white rectangle denotes the location of the fractured region.

**Figure 2** The density of the Marmousi2 model (in color) and the signal difference (in gray scale) for fractured and non-fractured models for the final time moment.
Conclusions

In this work the novel approach for the simulation of seismic waves in fractured transversely isotropic media was proposed. It is based on the grid-characteristic numerical method on rectangular grids. The method of splitting along the coordinate axes allows us to achieve the second order of accuracy. The standard Rusanov scheme for independent linear transport equations was used. To describe the presence of fluid-filled crack inside the VTI medium the computational mesh was split into node pairs. And correct contact conditions were set explicitly.

The in-house research software was developed and successfully tested on the complex Marmousi2 model. It was extended to the VTI case based on the empirical formulae for Thomsen parameters. The influence of the fractured object on the wavefield was demonstrated. This approach can be used to analyse seismic responses from more detailed and realistic geological models, rather than previously used. The next step of our research is to support arbitrary direction of fractures and three-dimensional problems.

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References


