Introduction

Seismic data generally have a wide scope of coverage and great detection depth, which is significant for oil and gas exploration. To uncover more geologic and sedimentary details from a 3D seismic volume, stratal slicing has been widely applied as an effective way for seismic imaging between different horizons (Zeng et al., 1998; Zeng, 2010). However, due to seismic noise and the interference effect among thin-beds, the prospecting targets can usually be ambiguous on stratal slices. For this, we propose a new method in this paper to highlight the geologic and sedimentary details on slices by using the multilevel 2D wavelet transform (Mallat, 2009). We first interpolate the slice with a size at least of $2^N \times 2^N$ to enable it to be N-level decomposed. The interpolated slice are then decomposed by applying a N-level 2D wavelet transform. We then reconstruct the decomposed data within different decomposition levels and select the optimal reconstructed slice which are the best among all the slices to uncover the geologic and sedimentary features of the target zone. The reconstructed slice is finally re-interpolated to the size of the original one. The effectiveness of the proposed method is validated by two examples.

Method and Theory

Stratal slices can generally be seen as an image of seismic data between different horizons, notated as $s(x, y)$ where $s(.)$ represents the seismic attribute displayed on the slice, and $(x, y)$ denotes the location. For an N-level 2D wavelet decomposition, we first interpolate the slice with a size at least of $2^N \times 2^N$, notated as $S(x, y)$ . 2D wavelet transform of $S(x, y)$ is then given as

$$FS_{W}(\sigma, \mu, \mu) = \frac{1}{\sigma} \int S(x, y) w(\frac{x-\mu}{\sigma}, \frac{y-\mu}{\sigma}) d\mu, d\nu,$$

(1)

where $FS_{W}(.)$ is the transform result and $w(.,.)$ denotes a 2D wavelet. The scale of $w(.,.)$ is controlled by $\sigma$, and the displacement in $x$ and $y$ direction by $\mu$ and $\nu$, respectively. The equation can be further simplified as $FS_{W}(\sigma, \mu, \mu) = \langle S(x, y), w_{\sigma, \mu, \nu}(x, y) \rangle$. For N-level 2D wavelet transform, the $i$-th ($1 \leq i \leq N$) level decomposition can thus be written as

$$
\begin{align*}
A'(x, y) &= \langle A^i(x, y), \phi_{iL} \rangle, \\
H'(x, y) &= \langle A^i(x, y), \psi_{iL} \rangle, \\
V'(x, y) &= \langle A^i(x, y), \psi_{iH} \rangle, \\
D'(x, y) &= \langle A^i(x, y), \psi_{iD} \rangle,
\end{align*}
$$

(2)

where $H(\cdot), V(\cdot)$ and $D(\cdot)$ denote the details coefficients matrices in horizontal, vertical, and diagonal direction, respectively. $A(\cdot)$ is the approximation coefficients matrix and $A'(x, y) = S(x, y) \cdot \phi_{iL}$ notates the 2D scaling function. $\psi_{iL}, \psi_{iH}, \psi_{iD}$ represent the corresponding wavelet functions, and $i \in [1, N]$. The decomposition results can be organized as $C = [A'(x, y), H'(x, y), V'(x, y), D'(x, y)]$.

The 2D inverse wavelet transform is given as

$$S(x, y) = \frac{1}{\sigma_{p}} \int_{0}^{\infty} d\sigma \int S_{W}(\sigma, \mu, \mu, \nu) w(\frac{x-\mu}{\sigma}, \frac{y-\mu}{\sigma}) d\mu, d\nu,$$

(3)

where $\sigma_{p} = \frac{1}{4\pi} \int W(\omega_{i}, \omega_{j})^{2} d\omega_{i} d\omega_{j}$, and $W(\omega_{i}, \omega_{j})$ is the Fourier transform of $w(x, y)$. Accordingly, the discrete form of a $k$ ($1 \leq k \leq N$) level reconstruction can be written as

$$R(x, y) = A'(x, y) \phi_{iL} + \sum_{m=1}^{N} \int_{0}^{\infty} [H'(x, y) \psi_{iL} + V'(x, y) \psi_{iH} + D'(x, y) \psi_{iD}] d\mu, d\nu,$$

(4)

where $\phi_{iL}, \psi_{iL}, \psi_{iH}$ and $\psi_{iD}$ denote the corresponding filters of $\phi_{iL}, \psi_{iL}, \psi_{iH}$ and $\psi_{iD}$, respectively. We finally re-interpolate the reconstructed slice $R(x, y)$ to the same size of $s(x, y)$, notated as $r(x, y)$ . By choosing an appropriate reconstruction level $k$ , the final result $r(x, y)$ can potentially be a better alternative to uncover more geologic and sedimentary details on the given slice. The corresponding residuals, notated as $e(x, y)$, can further be obtained as

$$e(x, y) = s(x, y) - r(x, y),$$

(5)
Example 1

A slice with strong noise is shown in Figure 1a. For vividly displaying the mechanism of multilevel 2D wavelet transform, we conduct a 2-level decomposition on the slice mentioned above. The first-level detail images in horizontal (H1), vertical (V1), and diagonal (D1) direction are shown in Figure 1b-1d, respectively. We further decompose the low-pass approximation and Figure 1c-1g are the second-level detail images (H2, V2, and D2). The second-level low-pass approximation (A2) is in Figure 1h.

For implementing a 10-level 2D wavelet transform, the slice is first interpolated with a size of 1024*1024 (2^10), and then decomposed in 10 levels by multilevel wavelet transform. We reconstruct the decomposed data within different levels, and finally re-interpolate the reconstructed slices to the size of 1024*1024. Slices reconstructed based on the last 1-R (R=1,...,10) level decomposed data are shown in Figure 2, sequentially. As we can see, the reconstructed slices with small reconstruction level R mainly show the low-frequency components of the original slice, and high-frequency components are partially filtered out. As R increases from 1 to 10, the difference between the original and reconstructed slices decreases monotonically. When R is set to 10, all the decomposed data are used to reconstruct a slice which is almost the same as the original one. In this example, we select the reconstructed slice with R is set to 8 and a comparison between the original and reconstructed slice is shown in Figure 3. Comparing to the original slice (Figure 3a), the reconstructed one (Figure 3b) reveals the channels and geologic details more clearly, as indicated by the arrows and ellipses.

The corresponding residual is also calculated (Figure 4a). A comparison between the 2D spectrum of the original (Figure 4b) and reconstructed (Figure 4c) slices indicates that, the high-frequency components of the original slice are partially filtered out. This is significant for repressing the random noise on the slice and thus increasing the accuracy of seismic interpretation.

![Figure 1](image1.png)

**Figure 1** a) A slice and its b-d) first-level horizontal (H1), vertical (V1), and diagonal (D1) detail images. The corresponding second-level detail images (H2, V2, and D2) are shown in e-g), respectively. And h) is the second-level low-pass approximation (A2).

![Figure 2](image2.png)

**Figure 2** Reconstructed slices based on the last 1-R level decomposed data.
Figure 3 A comparison between the a) original and b) reconstructed slices. Arrows and ellipses indicate the advantages of the proposed method.

Figure 4 a) Residuals and 2D spectra of the b) original and c) reconstructed slices. Black dashed lines indicate the differences in the 2D spectrum where low-frequency is near the centre, and high-frequency goes to the periphery.

Example 2

A 3D post-stacked seismic volume from Songliao Basin, northeast China is applied in this example. As we can see, a large anticlinal structure is observed in this area (Figure 5a). For better understanding the geologic details in this anticline, stratal slices are exploited to thoroughly investigate the sedimentary features in the target zone. Dashed lines in Figure 5a indicate the position of the stratal slice in Figure 5b in which channels are observed on both the flank of the anticline, but not clear enough for detecting the subtly sedimentary details.

Figure 5 a) Post-stacked 3D seismic volume from Songliao Basin, northeast China. Dashed lines indicate the position of the b) stratal slice of seismic amplitude.

To investigate the slice thoroughly, we first implement a 12-level 2D wavelet decomposition on the slice after interpolating it with a size of 4096*4096 (2^12). The decomposed data are then reconstructed within different levels. By analysing the reconstruction results, we choose the reconstructed slice with
R is set to 9, and re-interpolate it to the original size. As shown in Figure 6, the reconstructed slice (Figure 6b) not only shows the main channels (indicated by ellipses) more clearly and continuously, but also uncovers the small channels (indicated by arrows) which are almost indistinguishable on the original one (Figure 6a). A 3D display of the reconstructed slice is shown in Figure 7a. And in the residuals (Figure 7b), channels are observed as positive values, indicating a reinforcement of the channels on the reconstructed slice.

**Figure 6** A comparison between the a) original and b) reconstructed slices. Arrows and ellipses indicate the advantages of the proposed method.

**Figure 7** a) A 3D view of the reconstructed slice and b) the residuals between the original and reconstructed slices.

**Conclusions**

A new method for optimizing the stratal slices by using multilevel 2D wavelet transform is proposed in this paper. We first interpolate the slice with a size at least of $2^N \times 2^N$, and decompose the interpolated slice into N levels. The decomposed data are then reconstructed within different levels, and we further re-interpolate the best of all the reconstructed slice to its original size for uncovering more geologic and sedimentary details. Two examples in this paper have validate the effectiveness of the proposed method. We conclude that the proposed method here can significantly improve the potential of stratal slice in reservoir characterization. It can thus be a method of choice in seismic interpretation.

**References**

