Introduction

Achieving high quality results when migrating seismic data acquired in areas with complex near-surface geology can be challenging, owing to the heavy imprint the near surface can have on deeper sections of the image. A velocity model that accurately captures propagation in the shallow part of the subsurface becomes an important requirement for effective migration.

Tomographic inversion is a tool commonly used to generate such velocity models. A well-known shortcoming of tomographic methods based on refracted waves is that they can produce very inaccurate models in the presence of velocity inversions. Full waveform inversion (FWI) methods (Tarantola, 1984; Mora, 1987; Virieux and Operto, 2009) can overcome this limitation and boost the resolution, but the local optimization techniques typically used and the effect of noise make FWI a process difficult to implement, especially for land seismic data. For good quality results, an initial velocity model, sufficiently close to the ground truth, must be usually provided as input. An additional challenge is the high computational cost associated with 3D implementations of FWI.

In an effort to mitigate cycle-skipping issues related to the near absence of sub-5Hz frequency content in most seismic data, Shin and Ho Cha (2009) and Petrov and Newman (2014) propose carrying out FWI in the Laplace-Fourier domain. The extra exponential damping term present in their formulation acts as an extrapolator, extending spectral content to the lower frequencies.

In this paper, we propose a computationally efficient, easily parallelizable adaptation of Laplace-Fourier FWI for near-surface applications. Our implementation is based on the one described in Petrov and Newman (2014), modified for the acoustic case. Similar to Liu and Zhang (2013) and Aleardi and Mazzotti (2016), instead of inverting for a full 3D velocity model, we invert for separate 1D velocity profiles, under the assumption that the near-surface can be locally approximated by a 1.5D (i.e., laterally invariant, but vertically varying) velocity model. The collection of 1D velocity profiles can then be upscaled to a 3D velocity model, which can be further refined by a 3D version of FWI.

Method and Theory

We begin with the description of the forward modelling, which is used internally by the inversion. For acoustic 1.5D media that are isotropic and have constant density, the inhomogeneous Helmholtz equation in the Laplace-Fourier domain (Shin and Ho Cha, 2009; Petrov and Newman, 2012; Rivera et al., 2015) is given by

\[ \nabla^2 - k(z)^2 \tilde{u}(x, s) = \tilde{f}(x, s), \]  

(1)

in Cartesian coordinates \( x = (x, y, z) \). Here, \( s = \sigma + j\omega \) is the Laplace-Fourier complex frequency defined by a damping component \( \sigma \) and an angular frequency component \( \omega \), with \( j = \sqrt{-1} \). \( k(z) = s/c(z) \), where \( c(z) \) is the 1D compressional velocity profile defining the 1.5D acoustic medium and \( \tilde{f}(x, s) \), \( \tilde{u}(x, s) \) are the forcing term and modelled wavefield, respectively, transformed to the Laplace-Fourier domain. The forward transform from the time domain to the Laplace-Fourier domain is given by

\[ \tilde{u}(x, s) = \int_0^\infty u(x, t) e^{-st} dt. \]

In cylindrical coordinates, (1) becomes (Jensen et al., 2011, Eq. 2.35)

\[ [r^{-1} \partial_r r \partial_r + r^{-2} \partial_\theta^2 + \partial_z^2 - k(z)^2] \tilde{u}(r, s) = \tilde{f}(r, s), \]  

(2)

where \( \partial_\mu^n = \partial^n / \partial \mu^n \), \( r = (r, \theta, z) \), and \( r, \theta, z \) are the radial distance, azimuth and depth coordinates, respectively. Assuming that the source is omnidirectional, the wavefield \( \tilde{u}(r, s) \) is cylindrically...
symmetric (i.e., it has no azimuth dependence), due to the lack of lateral variation in the velocity model. Due to this symmetry the second term, \( r^{-2} \partial_{\theta}^2 \hat{u}(r,s) \), is zero and (2) simplifies to

\[
[r^{-1} \partial_r r \partial_r + \partial_z^2 - k(z)^2] \hat{u}(r,z,s) = \hat{f}(r,z,s). \tag{3}
\]

The Hankel transform allows us to take advantage of the cylindrical symmetry to further simplify (3). The zeroth order Hankel transform, \( b_0(\cdot) \), acting on the radial distance coordinate of a function \( \hat{g}(r,z) \), is defined as (Poularikas, 2010, Eq. 9.11)

\[
b_0[\hat{g}(r,z);r] = \int_0^{\infty} \hat{g}(r,z) J_0(rk_r)rdr = \hat{g}(k_r,z), \tag{4}
\]

where \( J_0(\cdot) \) is the zeroth order Bessel function of the first kind, \( k_r \) is the horizontal wavenumber and \( \hat{g}(k_r,z) \) is the function in the Laplace-Fourier-Hankel domain. Applying \( b_0(\cdot) \) to both sides of (3) and rearranging the terms yields

\[
[\partial_z^2 - (k_r^2 + k(z)^2)] \hat{u}(k_r,z,s) = \hat{f}(k_r,z,s), \tag{5}
\]

where we have used the property (Poularikas, 2010, Eq. 9.20)

\[
b_0[r^{-1} \partial_r r \partial_r \hat{u}(r,z,s);r] = -k_r^2 \hat{u}(k_r,z,s).
\]

The wavefield in the Laplace-Fourier domain can be then calculated with the aid of the inverse Hankel transform \( b_0^{-1}(\cdot) \) (Poularikas, 2010, Eq. 9.12),

\[
\hat{u}(r,z,s) = b_0^{-1}\{\hat{u}(k_r,z,s);k_r\} = \int_0^{\infty} \hat{u}(k_r,z,s)J_0(rk_r)k_rdk_r. \tag{6}
\]

The advantage of doing 1.5D modelling using (5) followed by (6) compared to directly solving (3) is that (5) is an ordinary differential equation that can be solved independently for each horizontal wavenumber \( k_r \). For our purposes, we solve (5) using a finite-difference method. The values of \( k_r \) at which to evaluate \( \hat{u}(k_r,z,s) \) affect the accuracy of the modelling and are determined through an adaptive sampling scheme.

In contrast to 3D versions of FWI where all input traces are inverted simultaneously to produce a full 3D velocity model, the proposed method divides data into shot gathers and inverts each gather independently. The result of inverting the \( i \)-th shot gather is \( m_i \), a local, discretized, 1D velocity model that approximates the true 3D velocity model. Each \( m_i \) is calculated by solving a regularized least-squares problem of the form

\[
m_i = \arg\min_{m} \left\{ \sum_{k=1}^{N} \|d_{\text{obs},i}(s_k) - L(m,s_k)\|_2^2 + w_{R}(m) \right\}. \tag{7}
\]

Here \( d_{\text{obs},i}(s_k) \) are the observed data from the \( i \)-th shot, evaluated at complex frequency \( s_k \). \( L(\cdot, s_k) \) models data at the corresponding receiver locations and frequency \( s_k \). \( R(\cdot) \) is a regularization term that encodes prior knowledge and/or promoting desired traits in \( m_i \). The scalar \( w_{R} \) is the associated regularization weight. In our case, \( R(\cdot) \) takes the form of Laplacian regularization, i.e.,

\[
R(m) = \|Dm\|_2^2,
\]

where \( D \) is a finite-difference approximation of the second derivative with respect to depth. This type of regularization is useful for producing smooth velocity profiles. The optimization problem (7) is solved using nonlinear conjugate gradients.
Regarding the frequencies $s_k$ at which to invert, we follow a strategy of progressively increasing bandwidth. We first invert at $s_1$. The resulting model is then used as an initial model for jointly inverting at $s_1$ and $s_2$, and so on. The complex frequencies $s_1, s_2, \ldots$ are ordered from low frequency/high damping to high frequency/low damping. This strategy takes advantage of the smoother shape of the objective function at low frequencies with high damping to avoid local minima in the beginning, while still making sure that the final model fits the data (in the least-squares sense) at all chosen frequencies.

The final 1D models can be upscaled to a uniform 3D grid, producing a velocity model that can be further used as an initial model for a 3D FWI.

Example

To test the feasibility of using the proposed inversion scheme for near-surface characterization, we performed a test based on synthetic data. The input data was synthesized using acoustic finite-difference modelling. The compressional velocity model used as input had the following features: i) a vertical gradient as background velocity (2475 m/s to 5000 m/s), ii) two velocity anomalies in the form of cubes, one with low (2725 m/s) and one with high (3775 m/s) average velocity and iii) a shallow high velocity layer (3750 m/s). A $y$-slice of the velocity model is shown in Figure 1a). A homogeneous density model with a value of 1000 kg/m$^3$ was used for modelling acoustic data.

The Laplace-Fourier input of the data was evaluated at frequencies ranging from 15 Hz to 50 Hz, with a step of 5 Hz. A constant damping of $7s^{-1}$ was used for all frequencies. First, the data at 15 Hz alone was inverted, using the background gradient as initial model. The velocity model produced by the inversion was smoothed and used as initial model for jointly inverting the data at 15 Hz and 20 Hz, etc., as per the progressive bandwidth increase strategy outlined in the method section. The receiver offsets in each inverted gather range from 25 m to 1475 m with a step of 50 m. The shot locations range from 500 m to 5500 m with a step of 100 m.

![Figure 1](image)

*Figure 1* a) The true model containing a low and a high velocity anomaly, embedded in a gradient with a shallow high velocity layer; b) the inversion result after lateral smoothing; c) comparison of the 1D velocity profiles at the locations marked with the dashed lines in a) and b).

The objective of the inversion test was to see if the proposed method can handle the velocity inversion occurring at the top of the model and whether the two anomalies could be recovered by the inversion. Note that this is not an “inverse crime” test: the input data were modelled by a 3D finite-difference modelling code, whereas the inversion code uses 1.5D forward modelling. The final resulting model can be seen in Figure 1b) for the same $y$-slice as in Figure 1a). Both the shallow high velocity layer and the two anomalies are clearly visible in the inversion result. This is further verified by comparing two
1D velocity profiles from the true model and the model produced by the inversion, shown in Figure 1c). The presence of edge artefacts around the cubes is expected to some extent, as the abrupt lateral change in velocity at those locations violates the 1.5D assumption. This can be partially mitigated in an upscaling/smoothing step, which enhances lateral continuity of the model in general. Further application of 3D FWI is also expected to correct edge artefacts, as propagation is handled more accurately.

Conclusions

Initial results on synthetic data indicate that the proposed 1.5D version of Laplace-Fourier FWI is able to handle velocity inversions and recover near-surface features, using as input data from 3D modelling. This suggests that a local 1.5D approximation can be a viable option for the purpose of creating an initial model for a 3D FWI. It is also an attractive option, as it allows to create such an initial model, avoiding the shortcomings of refraction tomography. The benefits of Laplace-Fourier FWI in dealing with cycle-skipping and its minimal preprocessing requirements are taken advantage of, without the excessive cost of a full 3D inversion.

Regarding implementation, as inversion for each 1D velocity profile can occur independently, a high degree parallelization is easy to achieve. Communication is required only for upscaling and/or for adding extra regularization that acts laterally. The method is currently being tested on real data showing promising results.

References


