Introduction

One of the most important parts of a seismic exploration may become the task of modelling the propagation of seismic waves in some specific regions. In many applications, the surface topology must be taken into account. Its influence includes secondary sources, appearing upon reflection of a wave from inhomogeneities at the boundary, wave diffraction and interference caused by surface or border structure.

The numerical methods for grid equations used to consider the different shapes of the realistic surface of the area under consideration can be divided into few base groups: making use of regular rectangular grids, utilizing adaptive meshes, and applying unstructured meshes. In the case of using unstructured meshes, the use of finite element methods gives good accuracy in a displacement calculation, but this entails great computational complexity increase and difficulties in constructing such suitable meshes [Tarrass et al., 2011]. The finite difference methods using adaptive meshes also allow a good description of the surface topology, however, because of the non-orthogonality of the mesh elements, particular near the borders, it is difficult to apply such boundary conditions as, for example, a free boundary or an absorbing boundary conditions [Gao et al., 2015].

In the paper presented here, we propose to consider the application of the overset grid method in combination with the grid-characteristic method (later referred to as GCM) [Favorskaya et al., 2018]. GCM has been studied in problems of modelling the propagation of seismic waves in other articles [Biryukov et al., 2016, Stognii P et al., 2019], and its high results of performing have been proved. The overset grid method requires a relatively small number of additional calculations, due to the main grid with a constant step Figure 1.d below the surface compared to rectilinear grid Figure 1.c. Furthermore, the overlying grid, in comparison with the cartesian cuts cell Figure 1.a, allows the use of various boundary conditions, moreover, it more accurately describes the surface than the staircase grid Figure 1.b.
Method

The idea behind the overset grid method is to utilize one or many additional grids to accurately describe the computation area features. This is performed by independent solving the equation in all grids and transferring the results between them afterwards using interpolation to eliminate the node-to-node match requirement.

Currently, our research only covers problem statements with non-changing physical domain properties, therefore it is possible to calculate some interpolation data during the grid preparation stage and later use the results obtained during the simulation to reduce the computation complexity. The methods we use consist of representing the solution function value in point as a weighted sum of known values in the other nodes. This approach can be seen in multiple interpolation methods, and as we do not impose any other restrictions any of them can be used. In practice, we use polynomial interpolation with bilinear and bicubic functions, but more advanced methods, such as natural neighbour interpolation can be used as well.

Theory

For an isotropic, homogeneous medium, neglecting all the forces except linear elasticity, we can write the equation of motion of the displacement of the elastic wave:

$$\rho \frac{\partial^2 U}{\partial t^2} = (\lambda + \mu) \nabla \cdot \mathbf{U} + \mu \nabla^2 \mathbf{U}$$  \hspace{1cm} (1)

The following abbreviations are called Lame’s parameters:

$$\lambda = \frac{E \nu}{(1 + \nu)(1 - 2\nu)}, \quad \mu = \frac{E}{2(1 + \nu)}$$  \hspace{1cm} (2)

The equation of motion for the longitudinal component of the displacement vector, which produce a primary wave:

$$\rho \frac{\partial^2 (\nabla \cdot \mathbf{U})}{\partial t^2} = (\lambda + \mu) \nabla \cdot (\nabla^2 \mathbf{U}) + \mu \nabla^2 (\nabla \cdot \mathbf{U}) = (\lambda + 2\mu) \Delta (\nabla \cdot \mathbf{U})$$  \hspace{1cm} (3)

The equation of motion for the transverse component of the displacement vector, which produce a secondary wave:

$$\rho \frac{\partial^2 \text{rot} \mathbf{U}}{\partial t^2} = \mu \nabla^2 \text{rot} \mathbf{U}$$  \hspace{1cm} (4)

Thus, the primary and secondary wave velocities, respectively:

$$C_s = \sqrt{\frac{E(1 - \nu)}{\rho(1 + \nu)(1 - 2\nu)}}, \quad C_p = \sqrt{\frac{E}{2\rho(1 + \mu)}}$$  \hspace{1cm} (5)

Where $E$ – is Young's modulus, $\nu$ – is Poisson’s ratio.

It is easy to see that for the two-dimensional case, the form of the equation of continuum mechanics for a linearly elastic model (1) will not change, and considering the transition to the two-dimensional definition of the divergence of the vector, we get that (3) takes the following form:

$$\rho \frac{\partial^2}{\partial t^2} \left( \frac{\partial U_x}{\partial x} + \frac{\partial U_y}{\partial y} \right) = (\lambda + 2\mu) \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \left( \frac{\partial U_x}{\partial x} + \frac{\partial U_y}{\partial y} \right)$$  \hspace{1cm} (6)

And the equation (4) turns into:

$$\rho \frac{\partial^2}{\partial t^2} \left( \frac{\partial U_x}{\partial x} - \frac{\partial U_y}{\partial y} \right) = \mu \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \left( \frac{\partial U_x}{\partial x} - \frac{\partial U_y}{\partial y} \right)$$  \hspace{1cm} (7)

Examples

To test the efficiency of our method, we conducted a series of test calculations for cases with complex topography. The result of one of the validation calculations with a curved surface is shown in Figure 2. In this problem statement, the waves have been propagated under a wavy earth surface defined by an overset grid (dark blue part of the figure) of sinusoidal shape. To numerically verify the result, exactly the same formulation of the problem was implemented for the “specfem2d” software package.
(http://www.geodynamics.org), which allows simulating the propagation of various types of seismic waves using the method of spectral elements on unstructured adaptive meshes [Komatitsch D., Tromp J., 1999]. Comparison of the methods was carried out by comparing the signals recorded at the receiver.

In this problem, a seismic wave propagates from a source at a depth of about 500 m below the surface, at a distance of 1000 m from it, at the same depth there is a receiver that records the derivative of the displacement been placed. The physical parameters of the medium under the free surface of the sinusoidal shape are as follows: \( C_s = 2500 \text{ m/s}, C_p = 1558 \text{ m/s}, \rho = 1500 \text{ kg/m}^3 \). The calculations were performed with a time step \( \Delta t = 200 \text{ ms} \) and a characteristic grid size \( \Delta x = 4 \text{ m} \), all the boundaries of the computational domain except the upper one (with a free surface condition) have absolutely absorbing boundary conditions.

As can be seen in Figure 3 the proposed method gives correct results and the differences in the recorded signal in both solutions are insignificant. It is worth noting separately that these differences are most significant in that part of the wavefront that was reflected from the free surface, which means that interpolation operations were performed 2 times on it (from the main grid to the overset one and then from overset into the main one), which explains the differences in the solution since interpolation is not a conservative operation in this implementation of the algorithm. However, the results converge quite well.

![Figure 2](image.png)

*Figure 2 This image is an illustration of the "wavy surface" problem statement and the resulting wavefield after simulation. Here green square is a source and yellow square is a receiver.*

**Conclusions**

In this paper, a modification of the grid-characteristic method for modelling the propagation of seismic waves in the presence of surfaces and boundaries with non-trivial topography is proposed. The proposed algorithm uses the overset grid method, which is a new approach for modelling such problems. Its advantages in comparison with alternative approaches are considered. A comparison is made with the method of spectral elements on unstructured meshes.

This method, using overset grids, can significantly simplify the process of creating a computational grid that would consider all the features of the computational domain of real physical problems. In addition, with this approach, a significant part of the calculations takes place on a regular rectangular grid, which allows us to carry out calculations more accurately and faster than when using unstructured meshes. There is also no need to build an unstructured meshes.
Method's practical applicability was verified using results, obtained by modelling physically identical processes of wave propagation under the sinusoidal surface using both proposed method and spectral element method. Despite a good accuracy, the method has a downside of being non-conservative. The work towards improving accuracy and three-dimensional problem-based comparison is currently in progress.

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Detector readings under curved surface

![Detector readings under curved surface](image)

**Figure 3** Comparison of the horizontal and vertical displacement derivatives between solutions of proposed method and specfem2d in the "wavy surface" problem statement

References


