Towards a 3D transdimensional ambient-noise surface wave tomography of Reykjanes Peninsula

Introduction

Seismic surface wave tomography is a well-known and popular method to obtain the shear wave velocity structure of the earth. Various methods and algorithms have been proposed for this purpose. The lack of a proper assessment of the model uncertainty, and the limitations arising from non-uniform data coverage are the main limitations of conventional linearized or iterative inversion schemes (Young et al. 2013). The transdimensional hierarchical Bayesian method introduced by Bodin et al (2012), which recovers 2D maps of surface wave velocity, addresses these two limitations. The transdimensional method is a Bayesian inference method that uses reversible jump Markov chain Monte Carlo (rjMcMC) and Voronoi cells to generate samples from the posterior probability distribution of the model given the observed data. Young et al (2013) and Galetti et al. (2017) used a two-step transdimensional scheme to recover the 3D surface wave velocity structure. The first step of this scheme involves the recovery of 2D maps of phase velocity using frequency-dependent travel time data at each frequency or period employing 2D transdimensional tomography method of Bodin et al (2012). This results in local (1D) dispersion curves (phase velocities which are frequency dependent). In the second step, each curve is locally inverted to estimate the depth dependent velocity model using a 1D transdimensional method. The estimated 1D depth dependent velocity models are then interpolated to build a 3D velocity structure of the subsurface. Zhang et al. (2018) showed that this two-step inversion scheme can be biased and also may cause the loss of information. Hence, they proposed a 3D transdimensional surface wave tomography method which uses a 3D discretization of the subsurface using Voronoi polyhedral tessellation which computationally has a comparable cost as the two-step transdimensional method yet preserving valuable information, better velocity structure and more interpretable uncertainty result (Zhang et al., 2018; Zhang et al., 2020). Here, we investigate the merits of this 3D inversion method for the recovery of the 3D shear velocity structure below the Reykjanes Peninsula. To that end, we generate synthetic data using the configuration of Reykjanese seismic array (RARR; Weemstra et al., 2021) and assuming a checker board synthetic velocity model. The frequency dependent travel times, which can be derived from phase velocity data, are then used in the fully 3D tomography algorithm of Zhang et al. (2020). However, we modified the algorithm in terms of prior probability definition and how to propose a sample to change the dimension of the model. The modification leads to a more efficient sampling of the posterior.

Ambient-noise surface dispersion data over Reykjanes Peninsula

The RARR consists of 83 seismic stations in the region of the Reykjanes Peninsula in southwest Iceland which have recorded seismic noise over a time span of about one year. The passive seismic data have been used to retrieve surface wave dispersion (phase velocities) using seismic interferometry by cross correlation. We aim to use the retrieved station-station dispersion curves for the purpose of 3D transdimensional tomography. Figure 1(a) shows the station locations available in the region depicted with yellow triangles. The distribution of stations is non-uniform, i.e. the station coverage is dense in one area while it is sparse in other areas. This implies that the achievable phase-velocity resolution can be expected to vary greatly across the region covered by the seismic array, higher in areas that are more densely covered by stations and decreasing where station density is low. For detailed information regarding the RARR, the field data, and the interferometric dispersion curves, we refer to Weemstra et al., 2021.

Forward function for calculating the surface wave dispersion data in a 3D earth model

Calculating the modeled surface wave dispersion data involves a number of steps (Zhang et al., 2018). By having a proposed 3D velocity model that is discretized using Voronoi polyhedral cells, we first consider a fine regular grid at the surface of the velocity model, then extracting the 1D velocity profile beneath each point, and finally calculating the associated (modeled) phase velocity dispersion curve associated with that point. These frequency dependent dispersion curves are calculated using a modal...
approximation method (Saito 1988; Herrmann 2013) for each of the depth-dependent 1D velocity profiles beneath each grid point. As such, we obtain 2D velocity maps of the surface wave at each frequency. These are subsequently used to compute frequency-dependent travel times, which is done using the fast marching method (e.g., Rawlinson & Sambridge, 2004).

3D transdimensional surface wave tomography

Transdimensional tomography method is based on Bayes’ rule, and uses Voronoi cells in conjunction with a reversible jump Markov chain Monte Carlo (rjMCMC) algorithm to allow the number of parameters to be variable. The rjMCMC algorithm involves different types of model perturbations. A particular choice of perturbation affects the convergence rate of the algorithm. Bodin and Sambridge (2009) used four different types of perturbation to efficiently sample the posterior distribution including velocity update, Voronoi cell move, death and birth. Bodin et al (2012) introduced also noise perturbation type to the algorithm to include inference of the data noise. These perturbation types allow the model to dynamically adapt itself to both data density and underlying velocity structure. For the 3D surface wave tomography, the 3D velocity structure will be perturbed in each step of the Markov chain according to one of the five aforementioned perturbation types. The surface wave dispersion data, which can be translated to frequency-dependent travel times, are then calculated to evaluate an acceptance probability as follows (Bodin and Sambridge, 2009):

\[
\alpha(m'|m) = \min \left[ 1, \frac{p(m')}{p(m)} \frac{q(m|m')}{q(m'|m)} \frac{p(d_{obs}|m')}{p(d_{obs}|m)} \right],
\]

where \(\alpha(m'|m)\) is probability of accepting proposed model \(m'\) given current model \(m\), \(\frac{p(m')}{p(m)}\) is the prior probability ratio of two models (current model \(m\) and proposed model \(m'\)), \(\frac{q(m|m')}{q(m'|m)}\) is the likelihood ratio of two models, \(\frac{p(d_{obs}|m')}{p(d_{obs}|m)}\) is the proposal ratio, and \(J\) is the Jacobian matrix, which accounts for (potential) differences in dimensionality between \(m\) and \(m'\) (i.e., a different number of cells). By assuming independent parameters, the prior can be written as (Bodin et al., 2012):

\[
p(m) = p(n)p(c|n)p(v|n)p(h),
\]

where \(n\) is the number of model parameters or cells, \(p(n)\) is the prior on the number of cells, \(p(c|n)\) is the prior on cell nuclei location, \(p(v|n)\) is the prior on cell velocity, and \(p(h)\) is the prior on noise hyperparameters. Following Bodin et al (2009) we assumed a uniform distribution, and defined a cubic area of the 3D seismic field where the nuclei of all the cells must lie with equal probability. Hence, the prior probability on cell nuclei location for the 3D model space reads:

\[
p(c|n) = \left( \frac{1}{\Delta x \Delta y \Delta z} \right)^n,
\]

where \(\Delta x, \Delta y, \Delta z\) are the difference between the maximum and the minimum allowable values for \(x, y, z\), respectively. Following Bodin and Sambridge (2009) the acceptance probability for the update and the move perturbation types read:

\[
\alpha(m'|m) = \min \left[ 1, \exp \left( \frac{\phi(m) - \phi(m')}{2} \right) \right],
\]

where \(\phi\) is the misfit function comparing the surface dispersion data to the calculated ones in the current model \(m\) and proposed model \(m'\). For birth and death steps, we used a uniform proposal kernel for proposing velocity in the new cell instead of a gaussian proposal. Hence, the acceptance probability for the birth step reads:

\[
\alpha(m'|m) = \min \left[ 1, \exp \left( \frac{\phi(m) - \phi(m')}{2} \right) \left( \frac{1}{n+1} \right) \right],
\]

where \(n\) is the current number of cells, and for the death step it will be:

\[
\alpha(m'|m) = \min \left[ 1, \exp \left( \frac{\phi(m) - \phi(m')}{{2^n}} \right) \right].
\]
The proposed new sample will be accepted or rejected based on this acceptance probability which is actually a modified version of the one proposed by Bodin and Sambridge (2009) for 2D travel time tomography.

Results and discussion

To investigate the feasibility of implementing the proposed 3D transdimensional method of Zhang et al. (2020) on the available surface wave dispersion data of Reykjanes, we designed a synthetic model in the same stations set up of the Reykjanes Peninsula. Figure 1(a) shows a checker board velocity model for the region within the available stations. The size of checkers is 10km and the higher velocity is 3km/s and the lower one is 2 km/s. The reliable range for picking the phase velocity in the field data is 0.1 to 0.5 Hz (Weemstra et al, 2021). Hence, we calculated synthetic phase velocities for all the receiver pairs in this frequency range. Then, these data are used in the fully 3D tomography algorithm. We compare the original algorithm of transdimensional tomography with the modified version described above for the recovery of 3D surface wave velocity structure. The initial velocity model, number of cells and noise parameter have been generated randomly with in the prior range of each parameter. We used three parallel chains of McMC and generated 50000 samples at each chain. The first 20000 samples have been discarded as the burn-in period of the McMC algorithm. Then we have taken one sample at every 100 samples to make sure there is no correlation between samples. Hence, we have 300 samples per chain, in total 900 samples. Figure 1(b) shows how the dimensionality of the model changes during three different chains of McMC algorithm. Figure 1(c-d) shows the post burn-in average of sampled models for the synthetic data generated using the original transdimensional tomography algorithm of Bodin and Sambridge (2009) and the modified algorithm proposed in this paper, respectively. As you can see, there is a good match between the true velocity model and the recovered model of the modified method. The recovered model is rather smooth that doesn’t need any further smoothing algorithm. The algorithm provides us with an uncertainty depicted in Figure 1(e-f) for the original algorithm and the modified version, respectively. The uncertainties depicted in Figure 1(e-f) are actually the variance of post burn in sampled models of the posterior.

![Figure 1](image.png)

**Figure 1.** Results of the one-step 3D transdimensional tomography. (a) Synthetic 3D checker board with the Reykjanes station locations depicted with black triangle. (b) Dimensionality change of model.
in three chains of McMC using the original transdimensional algorithm (blue lines) and the modified algorithm (red lines). (c-d) Mean velocity and standard deviation calculated from post burn-in samples generated using the original transdimensional algorithm. (e-f) Mean velocity and standard deviation calculated from post burn-in samples generated using the modified transdimensional algorithm proposed in this paper.

Conclusions

In this paper, we investigated the ability of 3D transdimensional tomography to recover the 3D surface wave velocity structure of Reykjanes area. We proposed some modification to the algorithm as well. Results showed that the proposed modifications improved the efficiency of the algorithm in order to recover the posterior distribution of the velocity model.

References