Bayesian linearized inversion of aspect ratio and fracture density based on seismic inversion

Introduction

Natural fractures are cracks or surfaces of breakage within rocks (Nelson 2001). By connecting pores together, fractures can greatly enhance permeability, and they are one of the major factors controlling fluid flow in tight reservoirs (Sava 2004). Therefore, successful development and management of fractured reservoirs (Nelson 2001) requires us not only to localize fractures but also to estimate their associated parameters, such as aspect ratio, fracture density and azimuthal orientation.

Many sources of information can be utilized to identify fractures in the subsurface, such as seismic data, well logs or core samples. Compared with well logs and core samples, seismic data or their inversion results could provide a larger 2D or 3D coverage over the target reservoir. In this paper, we use seismic inversion results to determine aspect ratio and fracture density for a quantitative characterization of fractured reservoir, in which a synthetic fracture model is built based on Hudson’s model (Hudson 1981).

Another objective of this paper is to address the challenge of uncertainty analysis in the prediction process. In particular, a Bayesian linearized inversion is presented (Grana 2016), where the goal is to obtain the posterior distribution of aspect ratio and fracture density. This new method is based on the linearization of Hudson’s model using a first-order Taylor series approximation (Grana 2016), which does not demand iterative optimization or sampling algorithm. Thus, the rock physical relation between elastic properties and fracture parameters is linearized, allowing the development of an analytical formulation of the inversion under Gaussian assumptions.

Hudson’s Model

According to Hudson (1981), the effective elastic moduli ($C_{ij}^{eff}$) of a fractured medium can be formalized as the following:

$$C_{ij}^{eff} = C_{ij}^0 + C_{ij}^1 + C_{ij}^2$$  (1)

where $C_{ij}^0$ is the elastic stiffness tensor of the isotropic background that is unfractured and the subscripts $i,j$ represents the 2-index Voigt notation for the stiffness matrix indices; $C_{ij}^1$ and $C_{ij}^2$ are the components of the first- and second-order corrections, respectively, which depend on the fracture parameters, such as aspect ratio and fracture density, as well as the bulk and shear moduli of the material filling the crack spaces (Sava 2004).

Hudson’s theory is well suited for different distributions of fractures developed in rocks, such as sets of parallelly, conically or randomly aligned cracks (Sava 2004). However, one limitation of this theory is that it cannot model fracture aspect ratios larger than 0.3 and fracture densities over 0.1. Additionally, with the second-order correction, for an increasing fracture density, the rocks do not fall apart, leading to unphysical analysis. Therefore, in the following study, only the first-order correction in equation (1) is used to compute the effective moduli of fractured rocks.

Bayesian Inversion

In inverse theory, a common approach is Bayesian inversion, in which the prior distribution of model parameters is combined with the likelihood to derive the posterior distribution of model parameters given the observed data (Tarantola 2005; Grana 2016). In this study, Bayesian inversion is applied to invert for the fracture parameters based on seismic inversion results, such as rock velocities and bulk density, and an estimation of uncertainty is given as credible intervals.
The forward rock physical model can be written as:

\[ d = f(m) + e \]  \hspace{1cm} (2)

in which \( d \) represent the rock properties, such as P- and S-wave velocities, and bulk density; \( m \) are the aspect ratio and fracture density defined in fractured rocks; \( e \) is the measured noise in the data or the imperfection of the physical model; \( f \) is the rock physical model that is typically non-linear, e.g. the Hudson’s model in equation (1). However, a linear approximation can be adopted for the non-linear model based on the Taylor series expansion, which can be formalized as the following:

\[ d \approx f(m_0) + J_{m_0}(m - m_0) + e \]  \hspace{1cm} (3)

where \( m_0 \) is the known value and is commonly assumed as the background or averaged value of \( m \); \( J_{m_0} \) is the Jacobian matrix of the rock physical model \( f \) that has been evaluated at the location of \( m_0 \) (Grana 2016).

Equation (3) can be further written as:

\[ d \approx J_{m_0}m + (f(m_0) - J_{m_0}m_0) + e = Fm + b + e \]  \hspace{1cm} (4)

And then:

\[ \bar{d} = d - b = Fm + e \]  \hspace{1cm} (5)

by subtracting the constant \( b \) from the observed data \( d \).

If the model parameters \( m \) are assumed to be Gaussian-distributed, and error \( e \) is also Gaussian with zero mean and covariance matrix of \( \Sigma_e \), then the posterior distribution of \( p(m|d) \) is Gaussian as well, and the posterior expectation and covariance can be computed as (Tarantola 2005; Grana 2016):

\[ \mu_{m|d} = \mu_m + \Sigma_m F^T (F \Sigma_m F^T + \Sigma_e)^{-1} (d - F \mu_m) \]  \hspace{1cm} (6)

\[ \Sigma_{m|d} = \Sigma_m - \Sigma_m F^T (F \Sigma_m F^T + \Sigma_e)^{-1} F \Sigma_m \]  \hspace{1cm} (7)

**Synthetic Study**

The proposed approach of Bayesian linearized inversion is applied to a synthetic model that is created from a thin-section micrograph (Figure 1). The vertical and horizontal axis have been enlarged to simulate an area of 200 × 200 m.

*Figure 1 Schematic view of the fractures on the cross section. The blue lines are traces of fractures and the red circles are pre-defined search areas for calculating aspect ratio and fracture density.*
The aspect ratio and fracture density are shown in Figure 2, and are to be inverted based on seismic inversion results.

Figure 2 (a) Aspect ratio and (b) fracture density, which are based on the fracture traces in Figure 1.

The unfractured background medium in Figure 1 is assumed to be dolomite, and the inclusion material within the crack space is gas. According to the first-order Hudson’s theory (equation 1), the rock properties in terms of bulk density, P- and S-wave velocities are shown in Figure 3, which are the inputs for the modelling of seismic data.

Figure 3 (a) Bulk density; (b) P-wave velocity; (c) S-wave velocity. These rock properties are used as inputs to synthesize seismic data.

PP pre-stack seismic data with 11 angle gathers are simulated with an incidence angle from 0 to 40°. A Ricker wavelet with a dominant frequency of 40 Hz is used as the source wavelet, and white random noise with a signal-noise-ratio of 30 dB is added to the seismic data (Figure 4, normal incidence). The fracture responses appear on the seismic data with negative amplitude (Figure 4), as the elastic parameters have been lowered by the presence of fractures and the infilling fluids (Figure 3). The fracture anisotropy is not considered in this study, which will be addressed in the future.

Figure 4 Normal incidence seismic data based on the rock properties in Figure 3 and a Ricker wavelet.
Based on the seismic data, a model-based deterministic elastic inversion scheme is applied to invert the rock properties, and the inverted P- and S-wave velocities are used as inputs for the prediction of fracture parameters. Specifically, the Bayesian linearized inversion approach proposed before is adopted for the quantification of uncertainty in the estimation of aspect ratio and fracture density. Figure 5a shows the inverted rock velocities at $X = 100$ m in the cross section (Figure 3), and the inverted fracture parameters are shown in Figure 5b, in which the 95% credible interval is calculated. Note that the computed bulk density is not used for the estimation of fracture parameters, as its inversion result generally has a lesser quality, compared to other rock properties (Feng et al. 2020).

![Figure 5](image)

**Figure 5** (a) Inverted P- and S-wave velocities at $X = 100$ m (Figure 3). (b) Computed aspect ratio and fracture density at the same location with predicted rock velocities as inputs.

### Conclusion

In this paper, we propose a Bayesian linearized inversion scheme to invert fracture parameters in terms of aspect ratio and fracture density. More specifically, the inverted rock properties such as P- and S-wave velocities are used as the inputs, which could exclude the location limitation from drilled logs and provide a wider coverage of the target reservoir. The conditional mean and credible interval of fracture parameters can be explicitly expressed and analytically calculated, as a Gaussian distribution is assumed, and a linearization of the Hudson’s model is performed.

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### References