Comparative study of stochastic Nature Inspired Optimization Algorithms to estimate Shear wave velocity using Ground Rolls.

Introduction

Rayleigh waves are dispersive and dependent upon the material’s elastic constant (Ugwu et al., 2018; Park et al., 1998a) and thus used in near surface geophysics to delineate the 1D shear wave velocity-depth models (Seshunarayana et al., 2012; Singh et al., 2019; Park et al., 1999), in engineering and environmental studies (Xia et al., 1999), to test the pavement (Ryden et al., 2004), in geotechnical studies to determine the stiffness of near-surface materials (Park et al., 1999; Long et al., 2020), in the classification of seismic site for evaluating the soil liquefaction and amplification potential (Foti et al., 2011; Olafsdottir et al., 2017).

Here we developed an algorithm to analyse datasets from an active, non-destructive and non-invasive geophysical method named Multichannel Analysis of surface wave (MASW) (Park et al., 1999; Seshunarayana et al., 2012; Long et al., 2020; Olafsdottir et al., 2017) to investigate and extract S-wave velocity-depth models from ground-rolls (Rayleigh waves). We have tested the developed algorithms on synthetic datasets. We used phase shift method to generate experimental fundamental mode dispersion curves (Park 2011) and open access codes (Olafsdottir et al., 2017) to generate the theoretical fundamental mode dispersion curves which we invert using two metaheuristic approaches: stochastic-based nature-inspired optimization algorithm, namely the Particle Swarm Optimization (inspired from the social and flocking behavior of a group of birds) (Poormirzaee et al., 2014; Song et al., 2012; Sengupta et al., 2019) and Grey wolf Optimization algorithms (inspired from the hunting and social behaviour of grey wolves) (Song et al., 2015; Mirjalili et al., 2014). We choose the global search stochastic optimization algorithms as these are independent of the initial model unlike the local search optimization algorithms (Song et al., 2012; Poormirzaee et al., 2014) and provide an optimal solution to the objective function iteratively from a range of possible solutions for different layers of shear wave velocities (Poormirzaee et al., 2014) by reducing the misfit or error between the experimental and theoretical dispersion curves (Olafsdottir et al., 2017). The goodness of the bestfit solutions from these inversion techniques is finally compared for the synthetic study.

Method and/or Theory

Methodology includes generation of experimental dispersion curves and their inversion which are explained as follows:

1. Experimental Dispersion Curve: - Dispersion curves are generated analysing the ground rolls (Rayleigh waves) from an active seismic shot gather data as follows:

   • Required Input Parameters: - (i) space-time domain data \( o(x_i, t) \), represents a multichannel seismic record which consists of equally spaced \( M \) channels in which the offset vector \( x_i \) is represented by: \( x_i = x_1 + (k - 1) \delta x \). where, \( \delta x \) represents the receiver spacing \( (k = 1, 2, 3, ..., M) \), (ii) testing of phase velocity vector, (iii) sampling interval and sampling frequency (Park 2011; Olafsdottir et al., 2017).

   • Fourier Transformation and Normalization of Amplitude: - The space-time domain data \( o(x_i, t) \) is transformed into a frequency domain as : \( F(x_i, w) = FFT\{o(x_i, t)\} \) (Park 2011; Olafsdottir et al., 2017), where, \( \omega = 2 \pi f \), is the angular frequency. The frequency domain data can be represented in the product form as: \( F(x_i, w) = A_k(w) \ast P_k(w) = A_k(w) \ast e^{(-i)\Phi_k(w)} \) (Park 2011, Park et al., 1998b; Olafsdottir et al., 2017) where, \( A_k(w) \) is the amplitude term and is basically influenced by the effect due to attenuation, source spectrum characteristics and due to spherical divergence, in contrast \( P_k(w) \) is the phase term and is influenced by the phase velocity of individual frequency and offset and is related to the dispersion property (Park 2011; Park et al., 1998b). The amplitude (\( F_{norm}(x_k, \omega) \)) can be normalized as follows (Park 1998b; Park 2011; Olafsdottir et al., 2017): \[
F_{norm}(x_k, \omega) = \frac{F(x_k, w)}{\text{mod}(F(x_k, \omega))} = P_k(w) = e^{(-i)\Phi_k(\omega)}
\]
where, \( \Phi_k(\omega) = (\omega \ast x_k)/c_\omega \).

- **Amplitude Summation operation**
  The amplitude summation operation is performed according to (Park 2011; Olafsdottir et al., 2017):-
  \[
  A(C_T, \omega) = \left( \frac{1}{N} \right) \sum_{k=1}^{M} e^{(-i)\Phi_k,T} F_{\text{norm}}(x_k, \omega) \tag{2}
  \]
  where, \( \Phi_{k,T} = \frac{\omega x_k}{C_T} \), is the phase-shift, and is determined by the testing of phase velocity \( (C_T) \) with a small increment between the max and min values of the velocity (Park 2011).

2. Inversion using Metaheuristic Approach

- **Particle swarm Optimization:** - PSO algorithm is inspired by the social behavior of a group of birds (Poormirzaee et al., 2014; Song et al., 2012). The inertia weight in the standard PSO algorithm were changed as: (a) by linearly decreasing the inertia weight \( w \) (Sengupta et al., 2019; Mirjalili, S.), (b) by using constriction coefficient (Sengupta et al., 2019; Mashayekhi et al., 2019; Poormirzaee et al., 2014), (c) by using damping factor (Mashayekhi et al., 2019)

- **Standard Grey Wolf Optimizer:** - GWO algorithm gets inspiration from the social and hunting behavior of grey wolf (Mirjalili et al., 2014; Song et al., 2015).

- **Improved Grey Wolf Optimizer:** - Mathematics of Improved GWO is similar to the standard GWO (as proposed by Mirjalili et al. 2014) but with a variable weight for updating the position (Gao et al., 2019).

In the above optimization algorithms, the misfit between the theoretical and experimental curves is calculated adapting from Olafsdottir et al., (2017) as follows:

\[
\text{Misfit}(\%) = \left( \frac{1}{N} \right) \sum_{i=1}^{N} \sqrt{\left( \frac{PV_{\text{experimental},i} - PV_{\text{theoretical},i}}{PV_{\text{experimental},i}} \right)^2} \times 100; \ N = \text{Total Number of observations}, PV=\text{Phase velocity Vectors of the Experimental and Theoretical Dispersion curve.}
\]

**Results**
The results of active MASW analysis are displayed in Figure 1. To gain confidence and to check the accuracy of the inversion results, we first generated a synthetic seismic shot gather (Figure 1a) with known properties of subsurface layers (Table 1) by using the forward modelling scheme of SOFI2D (Bohlen et al., 2015) for a two-layer velocity depth model overlying a half-space. The generated synthetic shot gathers (Figure 1a) with ground rolls contained in these were further imported into the MATLAB to generate the dispersion image (Figure 1b) with the fundamental mode and other higher modes using the equation 2 (Park et al., 1998b; Olafedottir E.A, 2017; Park et al., 2011). We then extracted the Rayleigh wave’s fundamental mode (Olafedottir E.A, 2017) by picking up the frequency and phase velocity values (Figure 1c) that were later on used for inversion using aforesaid PSO and GWO algorithms. The inversion results giving 1D shear wave velocity profile of all variants of PSO and GWO inversion algorithms are displayed in Figure 1d. The misfit value reduces to 0.197958 using constriction coefficient in the PSO algorithm. Comparing inversion results from PSO and GWO (standard and improved) variants yield errors of ~4.3 % for layer 1, ~4.5 % for layer 2, and ~2.5% for the half-space.

**Conclusions**
We analyze the ground-rolls (Rayleigh waves) from synthetic shot gathers using the Phase shift method and generate the dispersion images (phase velocity vs. frequency) from which we extract the experimental fundamental mode dispersion curves and invert these to obtain the 1-D shear wave velocity profiles of the near-surface materials using two stochastic optimization algorithms: PSO and GWO. Both of these algorithms provides bestfit solutions but the PSO algorithm converges to the minimum misfit values faster everytime than the standard and improved GWO algorithm. The developed algorithm will be tested on field datasets in a future publication.
Table 1: Parameters used in the generation of synthetic shot gather.

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<th>Parameters</th>
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Figure 1: (a) synthetic dataset, (b) dispersion image, (c) theoretical and experimental dispersion curves with misfit value, (d) inverted 1D Shear wave velocity Profile using PSO and GWO algorithm.

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References