Introduction

Improving the vertical resolution plays a significant role in seismic processing (Oliveira, Ferreira, Silva, & Brazil, 2019). Conventional methods are generally based on stationary convolutional model, which assumes that the seismic wavelet is time invariant (Robinson & Treitel, 1967). However, seismic wave propagates though a heterogeneous, viscous-elastic media of subsurface which brings both dispersion and attenuation effects, leading to the seismic wavelet’s variation with time, forming a non-stationary seismic data (Bickel & Natarajan, 1985).

Based on the study of Rosa and Ulrych (1991), the seismic wavelet spectrum is smooth and the reflectivity is statistically white. Thus, the time-varying wavelet can be estimated by smoothing the time-frequency spectrum of the non-stationary seismic data. Margrave, Lamoureux, and Henley (2011) proposed the Gabor deconvolution to estimate the time-varying wavelet by smoothing the time-frequency spectrum of Gabor transform, and then remove the effect of time-varying wavelet on non-stationary seismic data in Gabor domain. Gabor transform (Gabor, 1946), a time-frequency analysis technology, is used to analyse the non-stationary seismic data in Gabor deconvolution. In fact, one of the more popular technique is the generalized S-transform (GST) which avoids many shortcomings of the Gabor transform (Zhou, Wang, Marfurt, Jiang, & Bi, 2016).

In this paper, we begin expand GST to calculate the time-frequency spectrum of the non-stationary seismic data, and then convert that to logarithm time-frequency domain. Then, we estimate the time-varying wavelet spectral using Fourier series fitting in the logarithm time-frequency domain, which is key to improve vertical resolution. Finally, the time-varying wavelet spectral is used for balance seismic data to flatten the seismic response and improve vertical resolution. Our workflow is validated by a synthetic seismic data. Furthermore, a 3D coalfield seismic data is used for testing the performance of the method in enhancing the resolution.

Theory

Fourier transform establishes the relationship between time domain and frequency domain of signal. However, the time domain and frequency domain of Fourier transform are independent of each other, cannot represent the amplitude spectrum and phase spectrum at a specific time. To deal with the problem, Gabor (1946) proposed that the signal can be divided into different time windows, and each time window is analysed by Fourier transform, which can be written as

$$X(\tau, f) = \int_{-\infty}^{\infty} x(t)g(\tau-t)e^{-i2\pi f t}dt,$$  \hspace{1cm} (1)

where $X(\tau, f)$ means the time-frequency domain of the signal $x(t)$, $\tau$ is the parameter of time shift, and $g(t)$ is a Gaussian function (the window function), which is expressed as.

$$g(t) = \frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{1}{2}\left(\frac{t}{\sigma}\right)^2}.$$  \hspace{1cm} (2)

Thus, $x(t)$ can be expressed in a 2D time-frequency domain. However, once the parameter $\sigma$ of the window function was selected, the time resolution and frequency resolution are invariable for Gabor transform. If we denote

$$\sigma = \frac{1}{\lambda|f|^{p} + b},$$  \hspace{1cm} (3)

and substitute equation (3) and equation (2) into equation (1), we can obtain a GST

$$X(\tau, f) = \int_{-\infty}^{\infty} x(t) \left(\frac{1}{\lambda|f|^{p} + b} \right) e^{-\frac{(\tau-\lambda t)^2}{2(\lambda|f|^{p} + b)}} dt.$$  \hspace{1cm} (4)

It is obvious that the Gabor transform is a special case of the GST when $p=1, b=0$. The GST avoids the limitation of Gabor transform that the shape of the Gaussian window function is fixed, and we can adjust the parameters $\lambda, p, and b$ to meet our needs on analysing the non-stationary seismic data.

Method
Based on the GST, we can get the suitable time-frequency spectrum of the non-stationary seismic trace, which can be expressed as

$$X_n(t, f) = \log[\text{GST}(x_n(t))], \tag{5}$$

where $x_n$ is the non-stationary seismic trace, GST denotes the GST using the equation (4). log[.] means the logarithm. where $X_n(t, f)$ denotes the logarithm time–frequency spectrum.

In logarithm time–frequency domain, the amplitude spectral of the non-stationary seismic data is the sum of the amplitude spectrum of time-varying wavelet and the amplitude spectral of reflectivity. Due to the reflectivity is statistically white, we can smooth the amplitude spectral to estimate the time-varying wavelet in logarithm time–frequency domain. Inspired by Fourier analysis, the Fourier series is used to fit the amplitude spectrum of wavelet in logarithm time-frequency domain, which can be expressed as

$$\min \sum_{n=1}^{M} \varepsilon^2 = \sum_{n=1}^{M} |X_m(t, f) - F(t, f_m)|^2 \tag{6}$$

where

$$F(t, f_m) = \frac{a_0}{2} + \sum_{n=1}^{N} (a_n \cos(nk, f_m) + b_n \sin(nk, f_m)), \tag{7}$$

$\|\|_2$ denotes the squared $L_2$ norm of a function, $\varepsilon$ is the misfit error to be minimized, $t$ is the time shift parameter, $f$ is the frequency, $a_0, a_n$ and $b_n$ are the coefficient of Fourier series, $N$ is the order of Fourier series, $k$ is a positive constant. In fact, the $a_0, a_n$ and $b_n$ can be estimated by the nonlinear least squares method. The coefficient $a_0, a_n$ and $b_n$ of Fourier series are variant with time. Equation (6) represents Fourier series fitting method (FSF), which can estimate the time-varying wavelet in logarithm time–frequency domain.

After getting the estimated amplitude spectrum of time-varying wavelet, the phase of the estimated time-varying wavelet is determined by the minimum phase assumption. Finally, we design a deconvolution operator to finish deconvolution in time-frequency domain, and the deconvolution result $\tilde{X}_n(t, f)$ is expressed as

$$\tilde{X}_n(t, f) = X_n(t, f)O(t, f), \tag{8}$$

where $X_n(t, f)$ is the time-frequency spectrum of the non-stationary seismic data, $O(t, f)$ is the deconvolution operator. The time-domain trace can be reconstructing by the inverse GST. As a result, we proposed the Fourier Spectral Fitting (FSF) deconvolution method.

**Synthetic example**

Figure 2(a) is the reflectivity series, which is a standard to test the effect of deconvolution. Figure 2(b) is the non-stationary seismic trace which is the result of convolution of a minimum phase wavelet, the reflectivity series, and attenuation function ($Q = 50$). Figure 1(a) is the exact amplitude spectrum of the time-varying wavelet. Figure 1(b) is the fitting amplitude spectrum of the time-varying wavelet using FSF method. It is obvious that Figure 1(b) is similar to the exact amplitude spectrum of the time-varying wavelet. Figure 2(c) is the result of Gabor deconvolution for the stationary seismic trace. Figure 2(d) is the result of the FSF deconvolution method for the non-stationary seismic trace. Compared the Figure 2(c) and Figure 2(d), it is obvious that the FSF deconvolution method has higher resolution than the result of Gabor deconvolution, which indicates the good performance of the FSF deconvolution method.

**Field data example**

A coalfield 3D post-stack seismic data, acquired in east China, is used for testing the effectiveness of the FSF deconvolution method. Figure 3(a) shows the original seismic section (Inline=181), which is a vertical slice of the coalfield 3D post-stack seismic data. The Gabor deconvolution is used to process the original seismic section, and the result is shown in Figure 3(b). Figure 3(c) displays the result of the original seismic section after FSF deconvolution. It is obvious that the reflection events in Figure
3(c) have the highest vertical resolution than those in Figure 3(a) and Figure 3(b), especially the reflection events in the yellow box. Figure 3(d) shows the average spectrum of the seismic section in Figure 3(a)–(c), and the FSF deconvolution method can broaden the average amplitude spectrum effectively. Therefore, the FSF deconvolution method has better performance than the Gabor deconvolution method when processing the seismic section in this paper.

Conclusions

We propose a deconvolution method to estimate and remove the effect of time-varying wavelet on non-stationary seismic data. The GST is introduced to obtain the time-frequency distribution of non-stationary seismic data, and then convert that to logarithm time–frequency domain. In logarithm time–frequency domain, the non-stationary seismic data is the sum of the time-varying wavelet and the reflectivity. Due to the reflectivity is statistically white, we can smooth the amplitude spectral to estimate the time-varying wavelet in logarithm time–frequency domain. The higher-order Fourier series fitting is used for smoothing the amplitude spectral at every time sample of logarithm time–frequency domain. The test results of synthetic data show that the FSF deconvolution method can estimate time-varying wavelet with high precision, improve the vertical resolution, and compensate attenuation effects. Application to a coalfield 3D post-stack seismic data further demonstrates the proposed method is able to enhance the vertical resolution.

References


Figure 1 (a) Exact time-varying wavelet in time-frequency domain. (b) Estimated time-varying wavelet in time-frequency domain.
Figure 2 (a) Reflectivity, (b) Non-stationary seismic trace, (c) Non-stationary seismic trace after Gabor deconvolution. (d) Non-stationary seismic trace after FSF deconvolution.

Figure 3 (a) Original seismic section, (b) Original seismic section after Gabor deconvolution. (c) Original seismic section after FSF deconvolution. (d) Average amplitude spectrum.