Equivalent Q Estimation Using a Deep-learning-based Decoupling Method

Introduction

In seismic exploration, Q model estimation from post-stack seismic data is an important problem since this model is a key prerequisite for reservoir identification and improving the vertical resolution of seismic data. The effects of Q are mainly manifested as amplitude reduction and phase distortion of seismic data. In order to obtain high-resolution seismic data to describe oil and gas reservoirs, many Q factor estimation methods have been proposed. These methods can be roughly divided into direct estimation methods and inversion methods. Direct estimation methods, such as logarithmic spectral ratio method, frequency shift method, etc., usually use the attributes of seismic data to estimate Q, but they usually have disadvantages such as poor stability, dependence on source wavelet type, and the need for piecewise estimation of multi-layer Q model (Tonn, 1991). In contrast, the inversion methods regard Q factor as a model parameter and use the inversion algorithm to obtain the dynamic Q curve with traveltime or depth, which improves the calculation efficiency and stability, such as a novel method for Q analysis on reflection seismic data (Wang, 2004). However, the reflectivity and Q factor simultaneously affects the waveform of post-stack seismic data, leading to the fact that the Q model cannot be independently estimated without providing an accurate reflectivity model. The general approach for solving this problem is to simultaneously estimate these two parameters in an alternative iteration way (Wang et al., 2016). However, since neither the Q factor nor the reflectivity has a good initial model, the approach has no convergence guarantee.

In this paper, we propose a deep-learning-based decoupling method for building Q factor model. Specifically, we use deep learning to decouple the effects of two parameters (reflectivity and Q) on seismic data, and establish two new single parameter inversion problems using the deep-learning-based decoupled seismic data to independently and successively estimate reflectivity and equivalent Q. In order to effectively invert for the equivalent Q, we propose a new objective function with regularization terms and minimize it using the Levenberg-Marquardt (LM) algorithm. Compared with the alternate iteration method, a numerical example verifies the effectiveness of the proposed method.

Building Q model using alternate iteration method

The relationship among non-stationary seismic data, reflectivity and wavelet can be expressed in the frequency domain as (Bickel and Natarajan, 1985)

\[
\hat{s} = G r, \\
\text{s.t. } G_{jj} = \hat{w}(\omega_j) \exp[-i \omega_j t_i] \exp[-\omega_j t_i / 2Q_{\omega_j}] \exp[i \omega_j t_i \ln(\omega_j / \omega_r) / \pi Q_{\omega_j}].
\] (1)

where \( \hat{s} \) is the Fourier transform of the non-stationary seismic data; \( r \) is the reflectivity; \( G \) is the complex matrix with equivalent Q and source wavelet; \( Q_{\omega_j} \) is the equivalent Q; \( \hat{w}(\omega_j) \) is the Fourier transform of the source wavelet; \( \omega_j (j =1,2,\ldots,N) \) is the discrete angular frequency and \( t_i (i =1,2,\ldots,K) \) is the discrete two-way traveltime; \( \omega_r \) is the reference angular frequency: \( i = \sqrt{-1} \).

According to equation 1, the Q model cannot be built independently without providing the reflectivity. The general method is the alternate inversion of reflectivity and Q (or wavelet), then the two sub-problems of the \( k^{th} \) alternate iteration can be defined as

\[
r^{(k)} = \arg \min \left\| s_{\text{obs}} - G(Q^{(k-1)}_{\text{e}})r \right\|_2^2 + R_n(r) \] (2)

\[
Q^{(k)}_{\text{e}} = \arg \min \left\| \hat{s}_{\text{obs}} - G(Q_{\text{e}})r^{(k)} \right\|_2^2 + R_{\text{n}}(Q_{\text{e}}) \] (3)
where $\hat{s}_{\text{obs}}$ is the Fourier transform of the observed non-stationary seismic data; $k$ is the current number of iterations; $Rn_{1}(\cdot)$ and $Rn_{2}(\cdot)$ represent the regularizers of the two sub-problems, respectively.

### Building Q model using a deep-learning-based decoupling method

In this part, we introduce the proposed equivalent Q estimation method in detail. As introduced in equations 2 and 3, the alternate iteration method is very sensitive to the initial model, consequently, the convergence of the two inverse problems cannot be guaranteed. To solve this problem, we first use deep learning to correct observed non-stationary seismic data, which has been proved to be feasible (Gao et al., 2020). Here we choose a long short-term memory (LSTM) network to construct data correction operator $g(\cdot)$, which converts non-stationary seismic data into the corresponding stationary seismic data. The training set generated by three pairs of observation data and well logging data is used to train the LSTM network.

![Figure 1](image)

**Figure 1** The interval Q (a) and its corresponding equivalent Q (b).

Benefiting from the deep-learning-based correction operator $g(\cdot)$, we can decouple the effects of two parameters (reflectivity and Q) on seismic data. Consequently, we are able to firstly use the deep learning corrected data to independently estimate the reflectivity model and then estimate the equivalent Q model. Specifically, the following inverse problem is firstly used to estimate the reflectivity model:

$$
\mathbf{r}^* = \arg \min \|g(\mathbf{s}_{\text{obs}}) - \mathbf{W}\mathbf{r}\|_2^2 + \beta \|\mathbf{r}\|_1,
$$

where $\mathbf{r}^*$ is the optimal reflectivity; $\mathbf{s}_{\text{obs}}$ is the observed non-stationary seismic data in the time domain; $g(\mathbf{s}_{\text{obs}})$ represents the correction data generated by deep learning; $\mathbf{W}$ represents the wavelet convolution matrix formed by the source wavelet; $\|\mathbf{r}\|_1$ is a regularizer with sparse constraint on reflectivity; $\beta$ is the regularization parameter. It is worth noting that equation 4 can be solved without requiring the Q model. After solving the first inverse problem, we can obtain the reflectivity model which can then be used in the inversion of Q model. In the proposed method, the estimation of Q model is realized through the following inverse problem:

$$
\mathbf{Q}_e^* = \arg \min \|\mathbf{s}_{\text{obs}} - G(Q_e)\mathbf{r}^*\|_2^2 + \lambda_1^2 \|\mathbf{LQ}_e\|_2^2 + \lambda_2 \|\mathbf{L}_1\mathbf{LQ}_e\|_1 + \lambda_3^2 \|\mathbf{Q}_e - \mathbf{Q}_0\|_2^2,
$$

where $\mathbf{Q}_e^*$ is the optimal equivalent Q; $\mathbf{L}$ and $\mathbf{L}_1$ represent difference matrices of different dimensions; $\mathbf{Q}_0$ is the equivalent Q of the adjacent trace corresponding to the current trace; $\lambda_1, \lambda_2, \lambda_3$ are regularization parameters. As shown in Figure 1, $\mathbf{Q}_e$ can be approximately piecewise linear. The interval Q which does not change much corresponds to the flatness of $\mathbf{Q}_e$ curve. The interval Q which changes abnormally corresponds to the change in slope of $\mathbf{Q}_e$ curve. Therefore, the first regularizer $\|\mathbf{LQ}_e\|_2^2$ constrains the $\mathbf{Q}_e$ to be relatively flat. The second regularizer is the total variational (TV)
regularization, which is used to constrain the discontinuous jumps of the first derivative of \( Q_e \). The third regularizer guarantees the horizontal continuity of the inversion results.

In this paper, we firstly use the fast iterative shrink-threshold (FISTA) algorithm (Beck and Teboulle, 2009) to solve equation 4 and then use the Levenberg-Marquardt (LM) algorithm (Moré, 1978) to solve equation 5. The specific algorithm steps are summarized as follows:

Step 1: Using FISTA algorithm to invert for \( r^* \).
1) Equation 4 is given as a nonlinear inverse least squares problem that depends on decoupled data.
2) Let \( \beta = 0.05 \) and \( r^{(0)} = s_{\text{obs}} \).
3) Iteration of the FISTA algorithm.

Step 2: Using LM algorithm to invert for \( Q_e^* \).
1) Equation 5 is given as another nonlinear inverse least squares problem that depends on \( r^* \).
2) Let \( \lambda_1 = 0.3 \), \( \lambda_2 = 0.08 \), \( \lambda_3 = 0.05 \) and \( Q_e^{(0)} = 30 \) (The initial value of the equivalent \( Q \) is selected within a reasonable range of \( Q \) values).
3) Define the diagonal matrix \( F \) for each iteration to accommodate the non-differentiability of the 1-norm at some points.

\[
F_{ij} = \begin{cases} 1/|y_i|, & |y_i| \geq \varepsilon \\ 1/\varepsilon, & |y_i| < \varepsilon \end{cases}
\]

where \( |y_i| = |(LQ_e)_{j} - (LQ_e_{\text{true}})| \) and tolerance \( \varepsilon = 10^{-5} \).
4) Calculate the Jacobian matrix for each iteration \( \text{Jac} \) and substitute it into the LM algorithm.

\[
\text{Jac} = \left[ J, \lambda_1 L, \sqrt{0.5 \lambda_2} \sqrt{F} L, \lambda_3 I \right]^H,
\]

s.t. \( J_{ij} = \frac{-\omega_j t_i}{Q_e} \left[ -\frac{1}{2} + \frac{1}{2} \ln \left( \frac{\omega_j}{\omega_i} \right) \right] \hat{\omega} \left( \omega_j \right) \exp \left[ -i \omega_j t_i \right] \exp \left[ \frac{-\omega_j^2 / 2 + \ln \left( \omega_j / \omega_i \right)/\pi}{Q_e} \right] t_i \),

5) Iteration of the LM algorithm.

Numerical example

In this part, we will use a synthetic example to verify the effectiveness of the proposed method. This example is part of the modified Marmousi model, which has 600 traces and each trace has a length of 1.996 s. The true reflectivity and equivalent \( Q \) model of the modified Marmousi model are shown in Figure 2(a) and Figure 3(a), respectively. The Ricker wavelet with the peak frequency of 30 Hz is used herein to generate seismic data. Three pseudo wells, which are located at traces 150, 260 and 450, are used to construct training dataset for deep learning. In inversion, we respectively choose the observed seismic data as the initial model of reflectivity, and choose the constant vector \( 30 \) as the initial model of equivalent \( Q \).

Figure 2(b) and (c) respectively show the estimated reflectivity by the alternating iteration scheme and the proposed deep-learning-based decoupling method. Compared with the true reflectivity, the alternate iteration scheme fails to invert for the reflectivity after 1 s, and the PSNR is only 26.38 dB. This is because the scheme inverts for the reflectivity using non-stationary seismic data and it highly dependents on the initial model. After using deep learning to correct the non-stationarity, the estimated reflectivity significantly improved, especially in the middle and deep layers. The PSNR is 32.79 dB. This proves that the deep-learning-based decoupled seismic data can be used to effectively invert for reflectivity, which is beneficial to the next step of inverting the equivalent \( Q \).

Based on the estimated reflectivity, we further invert for the equivalent \( Q \) using the proposed method and compare it with the alternate iteration method in Figure 3(b) and (c). Clearly, the alternative iteration cannot estimate reasonable equivalent \( Q \) based on non-stationary data, and the PSNR is only 30.45 dB. The discontinuity in Figure 3(b) is because the inverted \( Q \) model is correct in the well locations but it is
poor in other places. On the contrary, the PSNR of our proposed method is 34.83 dB, the estimation result has been significantly improved and the low-Q abnormality can be estimated. These results verify the effectiveness of the proposed method and clearly demonstrate its superiority over the alternate iteration method.

**Figure 2** The reflectivity model: (a) is the true reflectivity model, (b) is estimated by alternating iteration method and (c) is estimated by the proposed deep learning-based decoupling method. The PSNR of (b) and (c) are 26.38 dB and 32.79 dB, respectively.

**Figure 3** The equivalent Q model: (a) is the true equivalent Q model, (b) is estimated by alternating iteration method and (c) is estimated by the proposed deep learning-based decoupling method. The PSNR of (b) and (c) are 30.45 dB and 34.83 dB, respectively.

**Conclusions**

In this paper, a deep-learning-based decoupling method is proposed for estimating equivalent Q model. By utilizing deep learning, we show that Q and reflectivity can be decoupled well. In addition, we established two new single parameter inversion problems to independently and successively estimate reflectivity and equivalent Q. The effectiveness of the proposed method was verified by the numerical example.

**References**