Introduction

Hydro-Mechanical (HM) coupling processes within shale reservoirs may significantly affect the gas production (Yan et al. 2018), and because shale matrix is the main gas storage space, the development of its HM model is important to the macroscopic simulations in shale gas reservoirs. At microscopic scale, shale matrix is composed of organic and inorganic matter, while the mechanical properties of these two media are quite different (Mehrabian & Abousleiman 2015), and both gas storage type and transport mechanism are also different in these two media (Song et al. 2016), thus we need to develop different microscale models to describe the coupling of gas flow and solid deformation in shale matrix. However, microscale models cannot be straightly applied to macroscopic simulations due to their huge calculation cost. Therefore, it is necessary to develop an efficient upscaling method to represent the microscale characteristics of organic and inorganic matter in macroscopic simulations.

Homogenization theory is usually used to upscale various physical processes in porous media from microscale to macroscale. It was first developed for solid mechanics, and then it was extended to describe the heat transfer and fluid flow in porous media (Auriault, 2010). Darabi et al. (2012) formulated a pressure-dependent permeability model, and applied homogenization theory to examine whether this model is preserved throughout the process of upsampling from local scale to large scale. Akkutlu et al. (2015) derived a homogenized model in the form of generalized nonlinear diffusion model to describe the effects of organic matter, and investigated different macroscale form of the homogenized model according to different gas transport regimes. Talonov and Vasilyeva (2016) studied the numerical homogenization for the gas transport in organic rich shale, in which the heterogeneous distribution of organic matter was considered, in this study, they compared the properties of fine-grid reference solution against those of coarse grid model, which was obtained by using the homogenized model (Akkutlu et al. 2015). Fan et al. (2019) developed an upscaled transport model based on homogenization theory, which can consider multiple transport mechanisms and the heterogeneity of shale samples, and then they employed their model to successfully explain pressure decay experimental data. To our best knowledge, the existing upscaling studies about shale gas are mainly focused on transport/flow problems, and the HM coupling is rarely involved.

In this paper, an efficient upscaling method based on homogenization theory is developed for the HM process in shale matrix, which can accurately represent the microscale characteristics of organic and inorganic matter in macroscale simulations. Firstly, shale matrix is assumed as a heterogeneous poroelastic medium composed of organic and inorganic matter, and according to different storage type and transport mechanism of real gas in these two media, the microscale HM model is developed. Specifically, the adsorption, viscous flow, Knudsen diffusion and surface diffusion are considered in organic matter, while only viscous flow and Knudsen diffusion are considered in inorganic matter. Besides, organic matter is characterized by low Young's modulus, and inorganic matter has high Young's modulus. After that, we assume that the mass fluxes in organic and inorganic matter are of similar order of magnitude, and then develop the following equivalent macroscale model for shale matrix based on homogenization theory:

\[
\nabla \cdot \sigma_{hm} = 0, \quad \sigma_{hm} = C^{eq}e(u^{hm}) - \alpha^{eq} \rho^{hm} I \quad (1)
\]

\[
\beta^{eq} \frac{\partial p^{hm}}{\partial t} + \frac{\partial m_{eq}^{hm}}{\partial t} + \alpha^{eq} \rho_{g} \frac{\partial (\nabla \cdot u^{hm})}{\partial t} + \nabla \cdot (\rho_{g} \nu^{hm}) = 0 \quad (2)
\]

\[
\nu^{hm} = -\left(\frac{k^{eq}}{\mu}\right) \nabla p^{hm} \quad (3)
\]

where the superscript hm means the homogenized shale matrix, and the equivalent parameters \( C^{eq}, \alpha^{eq}, k^{eq}, \beta^{eq}, \) and \( m_{eq}^{hm} \) are defined as
\[
C_{ijkl}^{\text{eq}} = \left( C_{ijkl} + C_{jmn} \delta_{im} \right), \quad \alpha^{\text{eq}} = \langle \alpha \rangle, \quad k_{ijkl}^{\text{eq}} = \frac{1}{|\Omega|} \int_{\Omega} \langle \alpha \rangle \, d\Omega
\]  
(4)

\[
\beta^{\text{eq}} = \rho_i \left( \frac{\alpha - \phi}{K_s} \right) + \langle \phi \rangle \frac{M_s}{RT} \left( \frac{1 - p}{Z} - \frac{\partial Z}{\partial p} \right), \quad m_{ij}^{\text{eq}} = \text{TOC} (1 - \phi) P_i P_{giz} V_i \frac{p/Z}{p_l + p/Z}
\]  
(5)

where TOC is the total organic content of shale matrix; the symbol \(<\cdot>\) stands for the volume average in shale matrix \(\Omega\). The \(\Omega\)-periodic vectors \(\omega\), of zero volume average over \(\Omega\), is the solution of following equations with periodic boundary

\[
\nabla_p \cdot \omega = 0, \quad \frac{1}{k_s(p)} \omega + \nabla \pi = e_i
\]  
(6)

where \(\pi\) is an unknown quantity in \(\Omega\); \(e_i\) is the standard Cartesian basis vector in \(i\)-th direction. Note that in Eqs. 4-6, \(p\) is considered as a parameter and these equations are solved for a fixed value of \(p\). Thus, \(k_{ijkl}^{\text{eq}}\), \(\beta^{\text{eq}}\), and \(m_{ij}^{\text{eq}}\) are computed for every value of \(p\), which gives us the look-up tables of these equivalent parameters. The \(\Omega\)-periodic vector \(\xi_{pq}\), of zero volume average over \(\Omega\), is the solution of following equations with periodic boundary

\[
\nabla_p \cdot \left( C_{ijkl} \tilde{e}_{ij} + e_{ij} (\xi_{pq}) \right) = 0, \quad \tilde{e}_{ij} = \frac{1}{2} \left( \delta_{ip} \delta_{jq} + \delta_{iq} \delta_{jp} \right)
\]  
(7)

where \(\delta_{ip}\) and \(\delta_{jq}\) are Kronecker symbols.

**Examples**

To verify the proposed method, we present a HM coupling problem, and compare fine grid reference solutions against those of the coarse grid with equivalent parameters. Schematic of the shale matrix (the dark parts represent organic matter) is depicted in Figure 1. The top boundary is permeable, on which the fluid pressure is 0.1MPa, and other boundaries are undrained. A uniform constant force is applied on the top boundary. The left and right boundaries are fixed in the \(x\)-direction and the bottom boundary is fixed in \(x\) and \(y\)-directions. Model parameters are given in Table 1. The pressure observation point and displacement observation point are located at the center of bottom and top boundaries respectively. Computational grids used for the fine grid and coarse grid are shown in Figure 1, and the equivalent parameters of each coarse grid is obtained according to Eqs. 4-7.

In Figure 2, the distributions of equivalent apparent permeabilities (at 1MPa) and equivalent elastic coefficient \(C_{1111}^{\text{eq}}\) are presented, we can see that organic matter has an important influence on the equivalent parameters and it is the main factor for the heterogeneity of shale matrix. The comparisons of pressure field and \(y\)-displacement field after 10s are shown in Figure 3, and we can see that the results of the fine grid and coarse grid are qualitatively close.

![Figure 1 Schematic of shale matrix (left, the dark parts represent organic matter) and computational grids for fine grid (middle) and coarse grid (right).](image-url)
Table 1  Model parameters of shale matrix

<table>
<thead>
<tr>
<th>Name</th>
<th>Value</th>
<th>Name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Porosity of organic matter</td>
<td>0.004</td>
<td>Langmuir pressure, MPa</td>
<td>13.79</td>
</tr>
<tr>
<td>Porosity of inorganic matter</td>
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<td>Langmuir volume, m³/kg</td>
<td>0.03</td>
</tr>
<tr>
<td>Young's modulus of organic matter, GPa</td>
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<td>Intrinsic solid grain bulk modulus, GPa</td>
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<tr>
<td>Young's modulus of inorganic matter, GPa</td>
<td>40.0</td>
<td>Shale matrix density, kg/m³</td>
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</tr>
<tr>
<td>Pore radius of organic matter, nm</td>
<td>2.51</td>
<td>Gas standard molar volume, m³/mol</td>
<td>0.024</td>
</tr>
<tr>
<td>Pore radius of inorganic matter, nm</td>
<td>25.1</td>
<td>Biot's coefficient</td>
<td>1.0</td>
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<tr>
<td>Molar mass of gas (CH₄), kg/mol</td>
<td>0.016</td>
<td>Tortuosity</td>
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<tr>
<td>Molecular diameter of gas (CH₄), nm</td>
<td>0.41</td>
<td>Poisson's ratio</td>
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<tr>
<td>Universal gas constant, J/(mol*K)</td>
<td>8.314</td>
<td>Formation temperature, K</td>
<td>323.0</td>
</tr>
<tr>
<td>Surface diffusion coefficient, m²/s</td>
<td>1E-8</td>
<td>Initial formation pressure, MPa</td>
<td>20.0</td>
</tr>
</tbody>
</table>

Figure 2  Equivalent permeabilities at 1MPa, and equivalent elastic coefficient for coarse grid.

Figure 3  Result comparisons of the fine grid (left) and coarse grid (right) after 10s.

Figure 4  Comparisons of pressure and displacement of two methods for different Langmuir volume.
Three different values of the Langmuir volume (Case 1, 0.01 m$^3$/kg; Case 2, 0.03 m$^3$/kg; Case 3, 0.05 m$^3$/kg) are investigated based on the fine grid and coarse grid. The comparisons of pressure and displacement from observation points for different cases are plotted in Figure 4, and the acceptable agreement between the results of fine grid and coarse grid for 3 cases can be seen. In addition, it also can be seen that the increase of Langmuir volume can reduce the pressure drop and y-displacement.

**Conclusions**

In this paper, an efficient upscaling method based on homogenization theory is developed for the HM process in shale matrix, which can accurately represent the microscale characteristics of organic and inorganic matter in macroscale simulations. Firstly, shale matrix is assumed as a heterogeneous poroelastic medium composed of organic and inorganic matter, and according to different storage type and transport mechanism of real gas in these two media, the microscale HM model is developed. Specifically, the adsorption, viscous flow, Knudsen diffusion and surface diffusion are considered in organic matter, while only viscous flow and Knudsen diffusion are considered in inorganic matter. Besides, organic matter is characterized by low Young's modulus, and inorganic matter has high Young's modulus. Then, the microscale HM model is homogenized to obtained the equivalent macroscopic HM model for shale matrix. Lastly, the accuracy of the proposed method is proved through the numerical example.

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**References**


