Pore structure based modeling of resistivity and inversion of saturation in fractured formation

Introduction

Fractured-vuggy reservoirs are one of the important areas for increasing oil and gas reserves and production at present and in the future. The lithology of fractured-vuggy reservoirs is complex, and the reservoir spaces are mostly porous, fractured, pore-vug, etc. Several types of reservoirs often exist at the same time. Due to the secondary reformation after diagenesis, the pore distribution often shows strong heterogeneity. Up to now, there is still no universally effective methods of evaluating the pore structure and the water saturation. Therefore, it is urgent to consider the resistivity-saturation model of pore structure to improve the accuracy of reservoir evaluation.

The logging evaluation of fractured-vuggy reservoirs mainly includes these aspects: the identification of fractured intervals, the quantitative evaluation of fractured-vuggy pore parameters and fluid-containing parameters. In 1985, Nelson's monograph "Geological Analysis of Natural Fractured Reservoirs" made the study of fractures transition from qualitative description to quantitative prediction. For the estimation of fluid saturation of fractured-vuggy reservoirs, the modified Archie formula is still used as the basis, using resistivity data combined with specific models. However, due to the complexity of the pore structure, sometimes Archie's formula does not apply. Over the years, many researchers have tried to find more effective reservoir saturation calculation models from different aspects such as pore structure and conductivity mechanism. Among them, Li Ning (1989) started from the theory of non-uniform anisotropic formation and its network conductivity, and gave a general relationship between resistivity and water saturation. The three-porosity model proposed by Aguilera (2004) is a representative model for estimating saturation of fractured-vuggy reservoirs. Liu Tangyan et al. (2006) used the sphere-cylinder model to evaluate the pore structure of rocks, combining the pore structure of the rock with the electrical properties of the rock, and refined the pore structure of different lithological rocks.

In this paper, based on the sphere-cylinder model, the resistivity response equation of fractured-vuggy reservoirs is constructed, combined with the real-space renormalization method proposed by King (1989), and the split-half method and Newton iterative inversion method are used to estimate the saturation of fracture-cavity reservoir. The anisotropic conductivity of each partition block was calculated by dividing the pore scale meshwork of the medium within the response range of the instrument at each logging point. Then, the conductance of the subdivision unit within the instrument response range at each logging point is renormalized to obtain the estimated resistivity response; finally, the simulated resistivity is compared with the measured resistivity, and the iterative bisect inversion method and the Newton iteration method are used to achieve the inversion of the water saturation of different subdivision blocks.

Method

Pore structure-resistivity model: In the actual logging exploration of fractured-vuggy reservoirs, we can usually obtain the porosity, resistivity data and approximate pore structure of the instrument response unit of each measurement point. The simplification of the response unit will bring great convenience to the study of the physical properties of fractured-vuggy reservoirs.

(1) Water-saturated pore resistivity model

As an abstract method of real rock pore structure, this study uses the sphere-cylinder model to simulate fractured-vuggy reservoirs. Without loss of generality, we use the cube grid block model. Each grid block contains a set of sphere-cylinder model. The partition block side length is \( h \), the radius of the inner ball \( r_x \) is \( \beta \) times the side length.
Figure 1 (a) A sphere-cylinder model with a three-path. (b-g) pore structure models with different pores and tubes.

Here we consider $C_d$, the ratio of tube radius $r_c$ and sphere radius $r_s$, and its value can be roughly determined according to the logging image. For simplicity, we can first consider the situation where the pores of the unidirectional partition block are saturated with water.

In the model, there are different $r_s$ value ranges, and the model shows different forms. In actual measurement, the porosity of fractured-vuggy reservoirs is usually within 36%. In the case of figure 1(d) and figure 1(f), the porosity is higher than 50%, so no need to think about it.

For the situation in figure 1(b), we can get the expression of pore volume and porosity:

$$\phi = \frac{4\pi}{3}r^3 + \pi \beta^2 C_d^2 (1 - 2 \beta \sqrt{1 - C_d^2}) - \frac{\pi}{3} (1 - \sqrt{1 - C_d^2}) \beta [2 \beta^2 C_d^2 + 2 \beta^2 (1 - \sqrt{1 - C_d^2})]$$  \hspace{1cm} (1)

Thus, we can obtain a cubic equation with one variable concerning $\beta$. By solving the equation, we can get $\beta$, and then get $r_s$ and $r_c$, so that the resistance, resistivity and conductance of the partition block can be obtained by integration:

$$R_0 = \left[ \frac{h - 2 r_s \sqrt{1 - C_d^2}}{\pi r_s^2 C_d^2} + \frac{1}{\pi r_s} \ln \frac{1 + \sqrt{1 - C_d^2}}{1 - \sqrt{1 - C_d^2}} \right] R_w$$  \hspace{1cm} (2)

(2) Oil-wetting and water-wetting pore resistivity model

For simplicity, we still consider the situation in the pores of unidirectional grid blocks. The partition blocks (side length is $h$) contains two sets of sphere-cylinder combinations, and the radius of the outer sphere is $r_s = \beta h$; the radius of the outer pipe is $r_c = \alpha h$; the ratio of tube radius to sphere radius is $C_d$. We suppose the radius of the inner ball is $r_{s0} = dh$; the radius of the inner tube is $r_{c0} = ch$; the ratio of tube radius to ball radius is $C_{d0}$.

Figure 2 (a-b) Two cases of water-wetting pore model. In the actual pores of the formation, the water layer is often evenly covered on the outside of the oil layer, and its thickness is relatively stable, so model in figure (b) is ignored here. (c) Oil-wetting pore model.

Taking the water-wetting model as an example, the saturation expression of the grid block:

$$S_w = \frac{V_w}{\phi h^3} = \frac{\phi d^2 [\pi c_1^2 + 4 \pi c_2^2] + 1 - \frac{1}{3} \sqrt{1 - c_2^2} \frac{4 \pi c_3 d^2}{3} - \frac{4 \pi c_4 d^2}{3} \sqrt{1 - c_2^2}}{\phi}$$  \hspace{1cm} (3)

We can get the $d$ by solving the equation, and then get $r_{s0}$ and $r_{c0}$, so that the resistance, resistivity and conductance of the partition block can be obtained by integration:
\[
R_t = \rho \frac{h-2r}{\pi r^2} \left[ 1 - \frac{c^2}{d^2} \right] + 2 \rho \int_{r_0}^{r_1} \left[ 1 - \frac{c^2}{d_0} \right] \frac{dr}{\pi \left( r_0^2 - r^2 - r_0^2 \right)} + 2 \rho \int_{r_0}^{r_1} \left[ 1 - \frac{c^2}{d_0} \right] \frac{dr}{\pi \left( r_0^2 - r^2 - r_0^2 \right)}
\]

(4)

**Saturation inversion method based on pore structure model:** In this study, for each characteristic response unit downhole, we can regard it as a system that needs to be renormalized, and we assume that the conductance of each partition block in the three orthogonal directions is different. We can obtain the resistivity of this system through renormalization to perform saturation inversion.

![Figure 3](a) Simplified 3D sphere-cylinder model and its effective resistivity. (b) Upscale of the grid system by renormalization process.

1. **Iterative inversion of saturation by half division method**

We enter a saturation value and then start the inversion. The partition block’s conductance can be obtained. Through renormalization, we can obtain the resistivity in three directions of the system, and the calculation result is compared with the actual observed resistance data rate to determine the saturation range, and iterate in turn to finally get an accurate saturation.

![Figure 4](The process of iterative bisect inversion of saturation.)

2. **Newton iterative inversion of saturation**

According to the previous derivation, we can express \( r_x \) as a function of \( S_w \). Expand \( \rho(S_w) \) at point \( S_{w,true} \) a Taylor, and derives the iterative formula:

\[
\Delta S_w = \left( \frac{\partial \rho}{\partial S_w} \right)^{-1} \Delta \rho
\]

(5)

\[
S_w = \Delta S_w + S_w^k
\]

(6)

**Examples**

We assume that each characteristic response unit is a 16×16×16 system, and take 2×2×2 primitive cells for calculation. Each primitive cell is composed of the same partition block, and the internal three-directional resistance of the block is different. The porosity of a partition block is 15%; in the x direction \( C_d \) is 0.1; in the y direction \( C_d \) is 0.15; in the z direction \( C_d \) is 0.2, and the true saturation is 24%. We input 40% and 20% respectively, and use the split-half method to iteratively invert the saturation, and observe the inversion results when the input value is higher or lower than the true value.
\[ \rho_{\text{cal}} = \rho_{x_{\text{cal}}} + \rho_{y_{\text{cal}}} + \rho_{z_{\text{cal}}} \quad \rho_{\text{obs}} = \rho_{x_{\text{obs}}} + \rho_{y_{\text{obs}}} + \rho_{z_{\text{obs}}} \]  

(7)

**Figure 5** (a-b) The iterative bisect inversion method shows a slow convergent rate, but do effectively reduce the resistivity error after 10 iterations with different inputs. (c) For different inputs, after 10 iterations, the saturation reaches a high accuracy.

**Figure 6** (a-b) Newton's method indicates fast convergent rate, and can quickly reduce the resistivity error for different inputs. (c) For different inputs, after 3 iterations, the saturation often arrived at extremely high accuracy.

**Conclusions**

In this study, the sphere-cylinder model combined with the renormalization method is used to simulate the medium within the response range of the logging tool, and the average saturation of the complex pore structure is predicted by the half-divided iterative inversion and Newtonian iterative inversion. The whole set of inversion methods have high accuracy and high practicability.

**References**


