Inversion of the reflected SH-wave for density and S-wave velocity structures

Introduction

Conventional seismic AVO inversion theories and methods are based on the propagation theory of PP wave. However, theoretical studies have shown that it is difficult to invert three physical parameters by using limited-offset PP wave seismic data (Tarantola, 2005). In seismic exploration, the estimation of rock density is very important for reservoir description and fluid interpretation. Compared with PP wave, reflected SS waves (SV-SV and SH-SH) are more sensitive to shear wave velocity and density. Compared with converted PS wave, SS waves are easier to process and the processing workflow for PP waves can be directly extended to SH waves. Therefore, inversion of SS waves can be helpful for better estimation of shear wave velocity and density.

With the development of horizontal vibrators technology, high-quality reflected SH-wave data is acquired from a 2D nine-component survey. In this paper, we present a simultaneous AVO inversion of the reflected SH waves based on a linearized approximation of the SH-SH wave reflection coefficient. Simultaneous inversion framework is applied to estimate shear wave velocity and density. Then we analyse the uncertainty and stability of our method by condition number and estimated model covariance. The performance of the SH-wave simultaneous inversion is demonstrated by using an SH-wave prestack dataset from a 2D nine-component survey.

The reflection coefficient of the SH-wave

For an isotropic subsurface model, an exact reflection coefficient of SH-SH waves at an interface can be written as (Aki and Richards, 1980)

\[
R_{SH} = \frac{\rho_1 V_{S1} \cos \varphi_1 \rho_2 V_{S2} \cos \varphi_2}{\rho_1 V_{S1} \cos \varphi_1 + \rho_2 V_{S2} \cos \varphi_2}.
\]  

(1)

where \( \varphi_1 \) and \( \varphi_2 \) represent the incident and transmitted SH wave angles, \( V_{S1} \) and \( \rho_1 \) denote the upper medium parameters, \( V_{S2} \) and \( \rho_2 \) denote the underlying medium parameters. We set \( V_s = (V_{S2} + V_{S1})/2 \), \( \Delta V = V_{S2} - V_{S1} \), \( \rho = (\rho_2 + \rho_1)/2 \), \( \Delta \rho = \rho_2 - \rho_1 \), and assume that \( |\Delta V/V_s| \ll 1 \) and \( |\Delta \rho/\rho| \ll 1 \). Then following Snell’s law, we have

\[
\sin \varphi_2 \approx \sin \varphi_1 \left(1 + \frac{\Delta V_s}{V_s}\right), \quad \frac{\cos \varphi_2}{\cos \varphi_1} \approx 1 - \tan^2 \varphi_1 \frac{\Delta V_s}{V_s}.
\]  

(2)

Then equation 1 can be expressed by ignoring higher-order terms as

\[
R_{SH} = -\frac{1}{2} \left( 1 - \tan^2 \varphi_1 \right) \frac{\Delta V_s}{V_s} + \frac{\Delta \rho}{\rho}.
\]  

(3)

Using the following expressions \( \ln \left(V_{s1}/V_s\right) \approx \Delta V_s/V_s \) and \( \ln \left(\rho_{s1}/\rho\right) \approx \Delta \rho/\rho \), the above equation can be further written as

\[
R_{SH} = -\frac{1}{2} \left( 1 - \tan^2 \varphi_1 \right) \ln \frac{V_{s1}}{V_s} + \ln \frac{\rho_{s1}}{\rho_1}.
\]  

(4)

This approximation enables a simultaneous inversion for model parameters \( V_s \) and \( \rho \).

We analyse the accuracy of the linear approximation (Eq.4) by comparing it with the exact reflection coefficient (Eq.1) (Fig.1). Table1 shows the elastic parameters of four single-interface models, two models (model 1 and 2) of moderate impedance contrasts and two models (model 1a and 2a) of strong impedance contrasts. Unlike the PP-wave that has varied AVO classes, there are only two types of SH-wave AVO as shown in Fig.1. As we can see, the linear approximation has high accuracy at 40° for moderate impedance contrasts and 30° for strong impedance contrasts. Therefore, the linear approximation can be used in the following inversion.
Table 1 Elastic parameters of four single-interface models

<table>
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<tr>
<th>Model</th>
<th>Layer</th>
<th>Vp (km/s)</th>
<th>Vs (km/s)</th>
<th>ρ (g/cc)</th>
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<td>Upper</td>
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<tr>
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<td>Upper</td>
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<tr>
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<td>Upper</td>
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<td></td>
<td>Lower</td>
<td>2.2</td>
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</tbody>
</table>

Figure 1 SH-wave reflection coefficients for four single-interface models. (a) and (b) correspond to Model 1 and 2, respectively (c) and (d) correspond to Model 1a and 2a, respectively.

Simultaneous inversion method for S-wave velocity and density

Model parameters can be solved by minimizing a least-square misfit function (Tarantola, 2005)

\[ J(m) = [g(m) - d]^T [g(m) - d] + \mu (m - m_{prior})^T (m - m_{prior}) \]

where \( m = [\ln V_S, \ln \rho]^T \) is the model vector, \( d = [d_1, \ldots, d_N]^T \) is the data vector consisting of seismic data for \( N \) angles, and \( g(m) \) is the nonlinear forward modelling operator including the exact SH-SH wave reflection coefficient. The second term is the regularization term, \( m_{prior} \) is the prior model vector. The nonlinear forward modelling can be linearized around the prior model \( m_{prior} \) by considering weak nonlinearity. So we can get the linearized objective function. The solution that minimizes the objective function can be obtained by differentiating the objective function \( J(m) \) with respect to \( m \) and equating the result to zero, then we can get the model parameter update \( \Delta m = m_{m_{prior}} \) is given by

\[ \Delta m = (G^T G + \mu I)^{-1} G^T \Delta d \]

where \( \Delta d = d - g(m_{prior}) \) is data residual, \( G \) is the linear operator and \( I \) is the identity matrix. We choose the Gauss-Newton method for iterative solution to get reliable inversion results.

Uncertainty analysis of SH-wave inversion

Firstly, we discuss the ill-conditioning of inversion problem by considering the linear forward modelling operator with eigenvalues and condition number.

\[ R = Fm \]

where \( R, F \) and \( m \) are reflection coefficient linear approximation, Jacobian matrix and model vector. The eigenvalues of the sensitivity matrix \( F \) are analysed by varying the maximum incident angle, as shown in Fig.2. As we can see, the first eigenvalue of PP-wave remains almost constant within the range of angles. Although the second eigenvalue of PP-wave is negligible for small angles, it increases rapidly with the increase of the maximum incident angle. It is close to the first eigenvalues when the maximum angle is larger than 30°. However, the third eigenvalue of PP-wave is still negligible for large angles. It is not close to the first eigenvalues until the maximum angle is larger than 60°. Compared with PP-wave, the second eigenvalue of SH-wave is close to the first eigenvalues when the maximum angle is about 35°. The condition number is calculated by the ratio of the first eigenvalue to the second eigenvalue. It shows that the inversion problem of PP-wave is ill-conditioned for middle incident angles.
due to high condition number (>50dB). With the increase of the maximum angle, the condition number of PP-wave is still higher than 20dB when the maximum angle is 60°. Compared with PP-wave, the condition number of SH-wave decreases rapidly and reaches 20dB when the maximum angle is 36°. Through analysis we can see that density is difficult to be inverted from PP-wave even at large angle (60°). Unlike PP-wave, S-wave velocity and density can be reliably inverted from SH-wave at moderate angle (30°-40°), because it is a well-conditioned inversion problem.

Figure 2 (a) Eigenvalues and (b) Condition number of PP-wave. (c) Eigenvalues and (d) Condition number of SH-wave. All are calculated on a logarithmic scale.

Then we use the model covariance matrix to analyse the uncertainty of the PP-wave inversion and SH-wave inversion. The model covariance matrix can be estimated as a function of data noise variance and the linear operator $F$ as $C_{in} = \sigma_n^2 [F^T F]^{-1}$ by assuming uncorrelated uniform noise (Downton, 2005). Figure 3 shows the uncertainty as a function of the maximum angle for PP-wave and SH-wave. It can be seen the model covariance matrix of PP-wave still has a high uncertainty when the maximum angle is 50°, especially for S-wave velocity and density. Compared with PP-wave, the model covariance matrix of SH-wave has a low uncertainty when the maximum angle is 40°. This implies that S-wave velocity and density can be well inverted from SH-wave at moderate angle (30°-40°). The above analysis shows that the SH-wave inversion is more stable than the PP-wave inversion, and can better invert the density and S-wave velocity at moderate angle.

Figure 3 Estimated model uncertainty based on the covariance matrix of the linear operator as a function of the maximum angle for PP-wave and SH-wave. (a)-(c) The maximum angles are 30°, 40°, and 50° of PP-waves. (d)-(f) The maximum angles are 30°, 40°, and 50° of SH-waves.

Field Data Example

A field dataset of SH-wave from a 2D nine-component survey is used to demonstrate the feasibility of our method. Reflected SH-SH waves were acquired in order to improve the image quality in gas cloud area. The SH-SH wave dataset within certain angle intervals are stacked to construct constant-angle sections (Figure 4). All the constant-angle sections from 10° to 30° are used to invert S-wave velocity and density. We use the calibrated well logs and interpreted horizons to generate the initial models with HRS software. Then we apply our inversion method to the SH-SH wave dataset, the inversion results are shown in Figure 5a and 5b. The inversion results at the well location are compared with the real logs, as shown in Figure 5c. Then we analyse the accuracy of the inversion results. The RMSE between the
real logs and estimated S-wave velocity is 3.57% and the RMSE between the real logs and estimated density is 4.32%. The cross-correlation coefficients corresponding to S-wave velocity and density are 0.91 and 0.86, respectively. It can be seen that our inversion results match well with the real logs. Therefore, our method shows a good performance in field-data application.

**Figure 4** Constant-angle sections of SH-wave for (a) 10°, (b) 20°, (c) 30°, The black lines represent the well location.

**Figure 5** (a) S-wave velocity inversion result, (b) Density inversion result, (c) Comparison between the real logs (black), initial models (grey) and inversion result at well location (red).

**Conclusions**

In this paper, we propose a simultaneous AVO inversion for SH-wave to estimate S-wave velocity and density, using the SH-wave linear approximation. Since SH-waves are more sensitive to density and S-wave velocity than PP-waves, inversion of SH-wave can get a better estimation of shear wave velocity and density. The stability and uncertainty analysis shows that the SH-wave inversion is more stable than the PP-wave inversion, and can better invert the density and S-wave velocity at moderate angle. A field data examples from a 2D nine-component survey have demonstrated that feasibility and accuracy of our proposed method. Besides, SH-wave is easier to process than PS-wave, our method has a greater potential for field application.

**Acknowledgements**

This work was supported by the National Natural Science Foundation of China (U19B6003), and the Strategic Cooperation Technology Projects of CNPC and CUPB (ZLZX2020-03). The vibroseismic data were provided by CNPC-BGP through the National Science and Technology Major Project (2017ZX05018005).

**References**

