Prestack data attenuation compensation based on inversion

Introduction

Due to the absorption of subsurface media, seismic waves experience energy attenuation and waveform distortion, which seriously decrease the resolution of seismic data. For prestack seismic data, since the effect of absorption attenuation varies with the propagation path, the amplitude variation with angle (AVA) trend will be distorted. Attenuation compensation is an effective method to eliminate the effects of absorption attenuation, but its stability and noise immunity have always plagued the application in the actual seismic data processing. To reduce these difficulties, research on attenuation compensation has been carried out by several groups. Wang (2002) develops a stable inverse Q filter method based on the wavefield continuation theory. Aiming at the instability of amplitude compensation, he applies the gain control method to improve the noise immunity. Wang (2011) derives an analytical equation of synthetic seismogram in attenuated media, regards the attenuation compensation as an inverse problem of Fredholm integral equation, and introduces Tikhonov regularization to improve stability. Li et al. (2015) study the attenuation compensation of prestack seismic records. Based on the different effects of offset and time components on absorption attenuation, they decompose the seismic records into two parts to compensate, and implement regularization constraints to improve the noise resistance. Zhang and Gao (2018) propose attenuation compensation based on synchronous squeeze wavelet transform. For the unstable phenomenon of amplitude compensation, they carry out constrained inversion based on L1 norm, which effectively controls the amplification of noise. Mehdi and Erik (2019) perform a local angle decomposition via the tau-p transform and apply Q compensation to the prestack marine data containing water-layer, which excellently resists the effects of incoherent noise and spatial aliasing.

In this paper, we consider the influence of different propagation paths on the absorption attenuation, improve the attenuation compensation method proposed by Wang (2011), and develop a novel stable prestack attenuation compensation method based on inversion. The proposed method can not only correct the amplitude attenuation and phase distortion, but also restore the AVA trend of the prestack gather. Numerical tests, comparative analysis of different compensation methods and noise immunity experiment demonstrate that the proposed method has higher accuracy and can perform attenuation compensation for prestack gather more stably and effectively.

Theory

Forward model

According to the exploding reflector idea, the harmonic wave exciting at depth time point $t_k$ is the product of the reflection coefficient at depth point $t_k$ and a series of simple harmonics $\hat{w}(\omega)e^{j\omega t}$:

$$s_i(t_i, \omega, t) = r(t_i)\hat{w}(\omega)e^{j\omega t}$$

(1)

Assuming that $h_k$ is the thickness between the depth $t_k$ and $t_{k-1}$, $\theta_k$ is the reflection angle, and $v(t_k)$ is the layer velocity, the harmonic waves of the depth $t_k$ propagating to the surface layer by layer forms the synthetic seismogram

$$s_2(t, \omega, t) = r(t)\hat{w}(\omega)e^{j\omega t}e^{-\sum_{i=0}^{2k} \frac{2h_i}{v(t_i)\cos\theta_i}}$$

(2)

We use the model described by Wang (2002) to introduce the attenuation, which replaces the velocity $v(t_k)$ in equation (2) with the following complex velocity

$$\frac{1}{v(t_k)} = \frac{1}{v(t_k, \omega)} \left(1 - \frac{j}{2Q(t_k)}\right), \quad v(t_k, \omega) = v(t_k, \omega_0) \left| \frac{\omega}{\omega_0} \right|^\gamma \gamma_0 = \frac{2}{\pi} \tan^{-1} \left(\frac{1}{Q(t_k)}\right) \approx \frac{1}{\pi Q(t_k)}$$

(3)

where $Q(t_k)$ is the quality factor of the depth $t_k$, $v(t_k, \omega)$ is the frequency-dependent phase velocity, and $v(t_k, \omega_0)$ is the reference velocity which generally is the phase velocity at dominant frequency $\omega_0$. Substituting equation (3) into equation (2), we can obtain
\[ s_i(t_i, \omega, t) = r(t_i) \tilde{u}(\omega) e^{i\omega t_i} \exp \left[ -i \omega \sum_{t_{i-1}}^{t_i} \frac{2h}{v(t_i, \omega_b) \cos \theta_1 Q(t_i)} \left| \frac{\omega_b}{\omega} \right|^n \right] - j \omega \sum_{t_{i-1}}^{t_i} \frac{2h}{v(t_i, \omega_b) \cos \theta_1 Q(t_i)} \left| \frac{\omega_b}{\omega} \right|^n \exp \left[ -i \omega \sum_{t_{i-1}}^{t_i} \left( \frac{\Delta t}{\cos \theta_1 Q(t_i)} \right) \left| \frac{\omega_b}{\omega} \right|^n \right] \]  

Taking into account that the prestack angle gather have implemented normal moveout (NMO) processing, therefore, the propagation time of the phase term still uses the zero-offset two-way travel time. Observing the equation (4), it is easy to see that the zero-offset two-way travel time from the depth \( t_k \) to \( t_{k-1} \) is \( \Delta t_k = \frac{2h_i}{v(t_k, \omega)} \), then the equation (5) is rewritten as

\[ s_i(t_i, \omega, t) = r(t_i) \tilde{u}(\omega) e^{i\omega t_i} \exp \left[ -i \omega \sum_{t_{i-1}}^{t_i} \frac{\Delta t_i}{\cos \theta_1 Q(t_i)} \left| \frac{\omega_b}{\omega} \right|^n \right] - j \omega \sum_{t_{i-1}}^{t_i} \frac{\Delta t_i}{\cos \theta_1 Q(t_i)} \left| \frac{\omega_b}{\omega} \right|^n \exp \left[ -i \omega \sum_{t_{i-1}}^{t_i} \left( \frac{\Delta t_i}{\cos \theta_1 Q(t_i)} \right) \left| \frac{\omega_b}{\omega} \right|^n \right] \]  

Taking the integral with respect to different reflection coefficients \( r(t_i) \) and frequency \( \omega \), we can get the equation for exploding reflection model in the frequency domain in viscoelastic media.

\[ \hat{s}(\omega) = \tilde{u}(\omega) \sum_{i=1}^{n} r(t_i) \exp \left[ -i \omega \sum_{t_{i-1}}^{t_i} \left( \frac{\Delta t_i}{\cos \theta_1 Q(t_i)} \left| \frac{\omega_b}{\omega} \right|^n \right) \right] \exp \left[ -i \omega \sum_{t_{i-1}}^{t_i} \left( \frac{\Delta t_i}{\cos \theta_1 Q(t_i)} \left| \frac{\omega_b}{\omega} \right|^n \right) \right] \]  

### Attenuation compensation

As we perform attenuation compensation, we assume that the seismic record has eliminated the effect of wavelet and the reflection angle \( \theta_1 \) of each layer can be calculated by ray tracing. Then, we define

\[ a(\omega, t_i) = \exp \left[ -i \omega \sum_{t_{i-1}}^{t_i} \left( \frac{\Delta t_i}{\cos \theta_1 Q(t_i)} \left| \frac{\omega_b}{\omega} \right|^n \right) \right] \exp \left[ -i \omega \sum_{t_{i-1}}^{t_i} \left( \frac{\Delta t_i}{\cos \theta_1 Q(t_i)} \left| \frac{\omega_b}{\omega} \right|^n \right) \right] \]  

Based on the above assumption, equation (6) can be reduced to

\[ \hat{s}(\omega) = \sum_{i=1}^{n} a(\omega, t_i) r(t_i) \]  

Equation (8) is a first kind Fredholm integral equation whose numerical solution is often unstable. Therefore, we utilize the Tikhonov regularization method to add a stability functional and construct the following functional

\[ OBJ = \left\| \sum_{i=1}^{n} a(\omega, t_i) r(t_i) - \hat{s}(\omega) \right\|^2 + \beta \Omega(r(t_i)) \]  

Where \( \beta > 0 \) is the regularization parameter, \( \Omega(r(t_i)) \) is the stability functionals including the function itself (zero-order regularization), the first derivative (first-order regularization) and the second derivative (second-order regularization) of the function. The expression is

\[ \Omega(r(t_i)) = \sum_{j=1}^{n} \left( r(t_i) \right)^j, \quad \Omega(r(t_i)) = \sum_{j=1}^{n} \left( \frac{dr(t_i)}{dt_i} \right)^j \quad \text{or} \quad \Omega(r(t_i)) = \sum_{j=1}^{n} \left( \frac{d^2r(t_i)}{dt_i^2} \right)^j \]  

To facilitate the solution, equation (10) can be discretized and rewritten in matrix form

\[ OBJ = \left\| AR - S \right\|^2 + \beta \left\| \Omega R \right\|^2 \]  

To minimize equation (11), we can achieve the attenuation compensation by the following equation

\[ R = (A^T A + \beta \Omega^T \Omega)^{-1} A^T \hat{S} \]  

### Examples

In this section, we first verify the effectiveness of our method through a geological model (see Figure.1(a)). We suppose that the wavelet is Ricker-like with a dominant frequency of 40 Hz, and take no account of geometric diffusion, transmission loss and NMO stretch. The reflection coefficients at

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different incident angles are calculated via the Zoeppritz equation. The range of incident angles is from 0° to 50°, with a discrete sampling interval of 1°. According to the equation derived above, we synthesize the prestack angle gather. Figure 1(b) and 1(c) show the prestack gathers without and with attenuation, respectively. We can find that the amplitude of the seismic wave is attenuated and the resolution is reduced in the vertical direction, and the AVA trend is distorted in the horizontal direction.

Figure 1 (a) A five-layer model including P-wave and S-wave velocity, density, and quality factor Q. (b) Prestack gather without attenuation. (c) Prestack gather with attenuation.

Figure 2 shows the compensation results using the poststack compensation method and our method, and the values of β are all set $1 \times 10^{-4}$. Through comparison, we can clearly see that the poststack compensation method has insufficient compensation in large-angle seismic traces because the influence of the propagation path is not considered. Advantageously, our three regularization methods can better compensate the amplitude and phase in large-angle seismic traces and improve the horizontal consistency of prestack gather.

Figure 2 The compensation results of different methods (a) Poststack compensation method based on inversion. (b) Zero-order regularization method. (c) First-order regularization method. (d) Second-order regularization method.

Figure 3 (a) The AVA curve for the bottom events in Figure 1 and 2. (b) The dominant frequency versus incident angle for the bottom events in Figure 1 and 2.
Simultaneously, the AVA curve and peak frequencies for the bottom events are plotted in Figure 3(a) and 3(b), respectively. From green line in Figure 3(a), it can be seen that the horizontal reflection characteristics are changed due to the influence of absorption. After compensation, our three methods can better eliminate the absorption difference related to the incident angle, and restore the AVA trend compared with the poststack compensation method. Also, it is explicit that the dominant frequency (green line) decreases with increasing incident angle (shown in Figure 3(b)), which weakens the horizontal consistency of the prestack gather. Through attenuation compensation, the dominant frequency can be restored without changing with the incident angle.

Finally, to examine the noise immunity of our method, 10% Gaussian random noise of maximum amplitude is added to the noise-free attenuation traces, which is shown in Figure 4(a). Figure 4(b)-4(d) correspond to the results using the zero-order, first-order, and second-order regularization methods, respectively. The results show that the three methods can stably compensate noisy data without significantly amplifying the noise, which further indicates that the two methods have better stability and noise resistance.

Conclusions

In this paper, we propose a novel prestack attenuation compensation method based on inversion considering the influence of ray paths on the absorption attenuation. We first derive the frequency domain forward formula of the prestack gather in the attenuation media, then reduce the attenuation compensation to an inverse problem, and utilize Tikhonov regularization for stability processing to achieve compensation. The synthetic and noise immunity experiment results illustrate that our method has high stability and accuracy when processing prestack data.

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References