Seismic migration acts as an adjoint operator of forward modeling. The resulting image is blurred and suffers from degraded spatial resolution, poor signal-to-noise ratio (SNR) and uneven amplitudes. Schuster and Hu (2000) use the migration Green’s function, also known as point spread function (PSF), to provide a fundamental understanding of the blurred image associated with a sequential application of the forward and adjoint operators. In detail, PSF physically describes the migrated results at the image space for the scattering point. It is noted that the blur is spatially varying due to nonuniform illumination. Perfect illumination makes the corresponding PSF quite sharp. The full seismic image is equivalent to weighted superposition of space-variant PSF; and the weight corresponds to the reflectivity of scattering point. Hence, seismic images follow a nonstationary convolution model between the space-variant PSF and the reflectivity.

Here we focus on the application of offset-dependent PSF to target-oriented prestack seismic image deblurring (Jiang and Zhang, 2019). For the prestack seismic image, convolution matrix generated by a series of nonstationary and poor PSFs is non-Toeplitz matrix with a large condition number, thus generating an ill-posed inverse problem. This leads to a huge computation cost in the iterative deconvolution method. Also, we need an extra regularization to make it stable. To cope with these problems, we propose a primal-dual optimization strategy combined with total variation (TV) regularization. Considering the nondifferentiability of TV, we obtain the dual formulation of TV norm that has a closed-form solution. Considering a non-Toeplitz matrix, which can not be implemented efficiently via the Fourier operator, we additionally dualize the data fitting term and avoid the expensive two-stage iteration. Finally, we transform the nonstationary deconvolution problem to the saddle-point problem, which can be solved efficiently via the extending primal-dual hybrid gradient (ePDHG) method (Chambolle and Pock, 2011). In particular, we use a diagonal preconditioner without the need to compute any step size parameters to accelerate iteration (Pock and Chambolle, 2011). Our proposed method is validated on both synthetic and field datasets, yielding the promising results in the prestack seismic image deblurring.

Method

Forward problem

Least squares migration (LSM) (Nemeth et al., 1999) approximates the inverse operator of forward modeling by minimizing the difference of the observed data and the predicted data in L2 norm:

$$\hat{r} = \min_r \frac{1}{2} \| Lr - d \|_2^2,$$

(1)

where $d$ denotes the vector of observed data, $r$ denotes the reflectivity model vector and $L$ denotes the forward modeling operator related to migration velocity, acquisition geometry and source wavelet. The normal equation of equation 1 reads

$$L^\dagger Lr = L^\dagger d = m,$$

(2)

where $L^\dagger$ represents the adjoint of forward modeling, namely migration operator; and $m$ denotes the migration image vector. Under the common-offset configuration, equation 2 establishes the forward problem of prestack seismic blurring, where $L^\dagger L$, namely offset-dependent PSF, acts as the blurring operator on offset-dependent reflectivity image $r$, resulting in a blurred prestack seismic image $m$.

Inverse problem

In pioneering works (Guitton, 2004; Valenciano et al., 2006; Lecomte, 2008; Jiang and Zhang, 2019), PSF calculation has been well performed. However, PSF is space-variant due to nonuniform illumination. Typically, PSF is not sharp enough and poor especially for the prestack case, resulting in strong off-diagonal components. To remove the migration artifacts entirely, we use the total variation (TV) norm to regularize the inverse problem:

$$\hat{r} = \min_r \mu \| L^\dagger Lr - m \|_2^2 + \| r \|_{TV}.$$

(3)
Here, the sign $\| \cdot \|_{TV}$ denotes the TV norm of input vector, and $\mu$ denotes the regularization parameter, which makes a compromise between data fitting and regularization.

We process the prestack seismic image block-by-block and suppose that the size of block is $n \times n$, and $r,m \in \mathbb{R}^{N \times 1}$, and $L^T L \in \mathbb{R}^{N \times N}$, where $N = n^2$. We describe the TV norm of reflectivity model vector $r$ as follows:

$$\| r \|_{TV} = \sum_{l=1}^{N} \| A_l^T r \|_2^2,$$

(4)

where $A_l \in \mathbb{R}^{N \times 2}$ and

$$A_l^T r = \begin{cases} (r_{l+1} - r_l, r_{l+n} - r_l)^T & \text{if } l \mod n \neq 0 \text{ and } l \leq N - n \\ (0, r_{l+n} - r_l)^T & \text{if } l \mod n = 0 \text{ and } l \leq N - n \\ (r_{l+1} - r_l, 0)^T & \text{if } l \mod n \neq 0 \text{ and } l > N - n \\ (0, 0)^T & \text{if } l \mod n = 0 \text{ and } l > N - n \end{cases}$$

(5)

Following the Cauchy-Schwarz inequality, equation 4 reads

$$\sum_{l=1}^{N} \| A_l^T r \|_2^2 = \max_{\| x \|_2 \leq 1} \sum_{l=1}^{N} x_l^T (A_l^T r)$$

$$= \max_{x \in \mathbb{R}} x^T A^T r$$

(6)

where $x_l^T = (x^T_1, x^T_2) \in \mathbb{R}^{1 \times 2}, x^T = (x^T_1, x^T_2, \ldots, x^T_N) \in \mathbb{R}^{1 \times 2N}$ and $x = \{x : x \in \mathbb{R}^{2N+1}, \|x\|_2 \leq 1 \text{ for } l = 1, 2, \ldots, N\}$, and $A = [A_1, A_2, \ldots, A_N] \in \mathbb{R}^{N \times 2N}$. Equation 6 defines the dual form of TV norm, and $x$ is the dual variable. Thus, we remove the barrier originating from the singularity of the TV norm via a dual variable.

Considering the nonstationary PSF, we introduce an extending dual variable $z \in \mathbb{R}^{N \times 1}$ to obtain the dual form of $\mu \| L^T Lr - m \|_2^2$, which reads

$$\mu \| L^T Lr - m \|_2^2 = \max_{\| x \|_2 \leq \mu} z^T (L^T Lr - m).$$

(7)

Substituting equation 6 and 7 into equation 3, we have

$$\min_{r} \max_{y \in \mathbb{Y}} z^T (L^T Lr - m) + x^T A^T r = \min_{r} \max_{y \in \mathbb{Y}} y^T (Kr - b),$$

(8)

where $z = \{x : x \in \mathbb{R}^{2N+1}, \|x\|_2 \leq \mu\}$, $y^T = (z^T, x^T) \in \mathbb{R}^{1 \times 3N}$, $K = \begin{bmatrix} L^T L \\ A^T \end{bmatrix} \in \mathbb{R}^{3N \times N}$, and $b = \begin{bmatrix} m \\ 0 \end{bmatrix} \in \mathbb{R}^{3N \times 1}$.

**Algorithm 1** the Extending Primal-Dual Hybrid Gradient (ePDHG) Algorithm

1. Initialize: $k = 0$, $r(k) \in \mathbb{R}^N$, $y(k) \in \mathbb{Y}$, $\sigma > 0$, $\tau > 0$, $\theta \in [0,1]$
2. while Not Converge do
3. $r(k+1) = r(k) - \tau K^T y(k)$
4. $y(k+1) = y(k) + \theta ((r(k+1) - r(k))$
5. $y(k+1) = y(k) + \sigma (Kr(k+1) - b)$
6. update $y(k+1)$
7. $k = k + 1$
8. end while

Equation 8 represents a typical saddle-point problem with the general form as

$$\min_{r} \max_{y \in \mathbb{Y}} F(r) + y^T Kr - G(y),$$

(9)

where $r$ is the primal variable with $F(r) = 0$ and $y$ is the dual variables with $G(y) = 0$. Hence, it is easy to implement the primal-dual hybrid gradient (PDHG) method (Chambolle and Pock, 2011) to solve
equation 8 via algorithm 1, where step 6 follows

\[
y_T^{(k+1)} = \left( z_T^{(k+1)} : x_T^{(k+1)} \right) = \left( \Pi_{L_2^0} (z^{(k+1)}) , \Pi_{L_2^1} (x_1^{(k+1)}) , \Pi_{L_2^2} (x_2^{(k+1)}) , \ldots , \Pi_{L_2^N} (x_N^{(k+1)}) \right).
\]

(10)

Here \( \Pi_{L_2^0} (\bullet) = \frac{\alpha}{\max \{ \| \bullet \|_2 ; \alpha \} } \) denotes the projection onto the ball with radius \( \alpha \).

Chambolle and Pock (2011) have proven convergence of the PDHG algorithm as long as \( \theta = 1 \) and the primal and dual step sizes, \( \tau \) and \( \sigma \) satisfy \( \tau \sigma L^2 < 1 \) where \( L = \| K \| \) is the operator norm of \( K \).

But, \( K \) is usually badly scaled and has a large \( L \) in algorithm 1. To cope with this problem, Pock and Chambolle (2011) have used a diagonal preconditioner instead of step size parameters selection to accelerate iteration. We adopt this approach, which replaces the step size \( \sigma \) and \( \tau \) in Algorithm 2 by the diagonal preconditioners:

\[
\Sigma = \text{diag} \left( \sigma_1, \ldots, \sigma_N \right), \quad \sigma_i = \frac{1}{\sum_{j=1}^{N} |K_{i,j}|^\alpha} \\
T = \text{diag} \left( \tau_1, \ldots, \tau_N \right), \quad \tau_j = \frac{1}{\sum_{i=1}^{N} |K_{i,j}|^{2-\alpha}}
\]

(11)

where \( \alpha \in [0,2] \) measures the amount of preconditioning in the primal and the dual variables, respectively. We adopt \( \alpha = 1 \). This way, we expect to take advantage of ePDHG algorithm combined with TV regularization to dublur the prestack seismic image.

Results

We first test the proposed method on the synthetic data with a constant velocity. Following (Jiang and Zhang, 2019), we calculates the analytical PSF independent of source wavelet, which means that the reflectivity model is convolved with source wavelet, as shown in Figure 1(a). We use the Kirchhoff 2D modeling method with analytical Green’s functions, introduced in Haddon and Buchen (1981), to generate the observed data. Figure 1(b) and 1(c) show the blurred common-offset section by migration with a deficient acquisition system and the deblurred common-offset section by our method. We observe that the proposed method removes migration artifacts entirely and recovers the reflection model.

Next, we present an field example of our method in Figure 2. Figure 2(a) and 2(b) show the blurred common-offset section by migration and the deblurred common-offset section by our method. Lots of migration artifacts are removed. We also present the blurred and deblurred common reflection point (CRP) gathers at 1900 cdp in Figure 2(c) and 2(d), respectively. We stack all traces in Figure 2(c) into together as the reference trace marked as red. Our results are more consistent with the reference trace. More importantly, we observe that event coherence is adopted and some weak events are probed plainly in Figure 2(d).

![Figure 1](image_url)

**Figure 1** Synthetic data. (a) Reflectivity model, (b) blurred common-offset section by Kirchhoff migration, and (c) deblurred common-offset section by our method.
Conclusion

We have developed an alternative prestack seismic deblurring method regularized by total variation (TV) norm. The dual forms of TV term and L2 fitting term are derived to obtain the closed-form solution of primal-dual hybrid gradient (PDHG) method. Thus, the prestack seismic deblurring problem can be efficiently resolved. The validity of our method is fully demonstrated by synthetic and field data.

Figure 2 Field example. (a) Blurred common-offset section by Kirchhoff migration, and (b) deblurred common-offset section by our method. (c) Blurred common reflection point gather by Kirchhoff migration, and (d) deblurred common reflection point gather by our method at 1900 CDP. We stack all traces in (c) into together as the reference trace (red).

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