3D inversion of airborne EM data with topography

Introduction

Working on a moving platform, the time-domain airborne EM system can conduct rapid 3D detection over mountainous areas where the ground EM survey is very difficult to carry out due to rugged topography. Now, it has been widely used in mineral resources exploration, groundwater and engineering environment (Smith et al., 2010; Yin et al., 2015). In the past thirty years, the geophysicists have been working on the conductivity-depth imaging and 1D inversion techniques (Macnae et al., 1998; Vallee, and Smith, 2009) based on a layered earth model. These kinds of methods can obtain ideal imaging results in flat terrain area. However, due to the serious impact of topography on airborne EM data (Yin et al., 2016), it may meet some problems when these methods are applied to topographic data. The data can be misinterpreted as the topographic responses are taken as useful EM signal, which can produce serious influence on the interpretation accuracy of time-domain airborne EM data. Therefore, developing a 3D inversion algorithm for a topographic earth model is very important for AEM data interpretation.

In this paper, we develop a 3D topographic inversion method for time-domain airborne EM data based on unstructured mesh. For the forward modelling part, we use the unstructured time-domain finite-element algorithm developed by Yin et. al. (2016) and Qi et. al. (2017). While for the inversion part, we adopt the Gauss-Newton method. We first subdivide the topographic earth with unstructured tetrahedron mesh to construct an inversion mesh. Then, the forward modelling and inversion meshes are decoupled based on the moving footprint and unstructured local mesh. We quickly calculate the forward responses and sensitivity matrix on the local forward modelling mesh in combination with parallel technology. Finally, Gauss-Newton method is used to complete the time-domain airborne EM 3D inversion. Our inversion method is applied to synthetic data inversion to verify its correctness.

Method

1. Forward modelling based on unstructured finite-element method:

We start from the electric field diffusion equation

$$\frac{1}{\mu} \nabla \times \nabla \times e(r,t) + \sigma \frac{\partial e(r,t)}{\partial t} + \sigma \frac{\partial j_s(r,t)}{\partial t} = 0$$

(1)

where $e(r,t)$ is the electric field at time $t$ and position $r$, $\mu$ and $\sigma$ are respectively the magnetic permeability and conductivity, while $j_s(r,t)$ is the imposed current source. We use the vector basic functions to discretize the diffusion equation in space for the electric field and express the electric field in each element as

$$e(r,t) = \sum_{j=1}^{6} u_j(t) n_j^i(r)$$

(2)

where $u_j(t)$ is the electric field on $j$-th edge of $k$-th element. $n_j^i(r)$ are the vector basic functions that satisfy the divergence-free property and tangential continuity of the electric field. By using the Galerkin’s method, we can establish the finite-element governing equation of

$$\frac{de(t)}{dt} + Se(t) + J = 0$$

(3)

where $M$ and $S$ are respectively the mass and stiffness matrices, $J$ is the source term. For each element, $M$, $S$ and $J$ are given by

$$M_{ij}[i,j] = \int \sigma \left[ n_i^j(r) \cdot n_j^i(r) \right] d\nu$$

$$S_{ij}[i,j] = \int \left[ \mu \nabla \times n_i^j(r) \cdot \nabla \times n_j^i(r) \right] d\nu$$

(4)

$$J_{ij}[i,j] = n_i^j(r) \cdot u \frac{dl_s(t)}{dt}$$

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where \( \mathbf{u} \) denotes the direction of the current flow, \( l \) is the length of the electrical dipole crossing the tetrahedron element, \( i, j = 1, 2, \ldots, 6 \). We apply the second-order Backward Euler scheme to equation (3) and obtain the following equations system of second-order implicit difference, i.e.

\[
(3M + 2\Delta t \mathbf{S}) \mathbf{e}^{m+2}(t) = M(4\mathbf{e}^{m+1}(t) - \mathbf{e}^{m}(t)) - 2\Delta t \mathbf{J}^{m+2}
\]

where \( \mathbf{e}^{m}(t) \) denotes the electric field for \( m \)-th time channel and \( \Delta t \) is the time step.

2. 3D inversion theory for time-domain airborne EM:

We first establish the following objective function

\[
\Phi(\mathbf{m}, \mathbf{d}_{\text{obs}}) = \frac{1}{2} \| W_{d}(F(\mathbf{m}) - \mathbf{d}_{\text{obs}}) \|_{2}^{2} + \frac{\lambda}{2} \| W_{m}(\mathbf{m} - \mathbf{m}_{\text{ref}}) \|_{2}^{2}
\]

where \( \mathbf{m} \) is an \( M \)-dimensional model vector, \( \mathbf{d}_{\text{obs}} \) is an \( N \)-dimensional vector of observed data, \( W_{d} \) and \( W_{m} \) are respectively the data and model variance matrix, \( F \) is the forward operator, \( \mathbf{m}_{\text{ref}} \) is the reference model, \( \lambda \) is the trade-off parameter. Then, we adopt the Gauss-Newton method to establish the inversion equation

\[
\left( J^{T} W_{d}^{T} W_{d} J + \lambda W_{m}^{T} W_{m} \right) \Delta \mathbf{m} = -g(\mathbf{m})
\]

where \( J \) is the Jacobian matrix and the gradient of objective function \( g(\mathbf{m}) \) is given by

\[
g(\mathbf{m}) = J^{T} W_{d}^{T} W_{d} (F(\mathbf{m}) - \mathbf{d}_{\text{obs}}) + \lambda W_{m}^{T} W_{m} (\mathbf{m} - \mathbf{m}_{\text{ref}})
\]

Finally, by using the conjugate gradient method to solve equation (8), we can obtain the model update \( \Delta \mathbf{m} \) and update the model.

**Example**

To check the reliability of our 3D inversion codes, we design a topographic model as shown in Figure 1. The topography has a maximum rise of 100m. Two conductive abnormal bodies of 10 \( \Omega \cdot \text{m} \) are buried in a 100 \( \Omega \cdot \text{m} \) half-space. The width of the dipping plate is 40m along the y- direction, 400m along the x-direction, and 250m along the z-direction. Its top depth is 60m. The conductive cube with a dimension of 150m×150m×150m is buried directly under the peak, with a top depth of 150m. The survey system is a central loop configuration. The transmitter is a 5-turn regular 12-sided polygon with a side length of 9m. The transmitting current is shown in Figure 2. The airborne EM system keeps a constant clearance of 30m to the earth surface along the survey lines. The survey stations distribute uniformly in the whole survey area with 40m spacing, yielding 21 × 21 = 441 survey stations. The observation time ranges from 40μs to 5ms, we have totally 10 logarithmically equal time channels. The dBz/dt responses contaminated by 5% white Gaussian noise are used as data in our inversion. The numerical experiments are performed on a cluster with 16 computer nodes and 32GB memory for each node.

![Figure 1 Inversion model with two conductive bodies of 10 \( \Omega \cdot \text{m} \) buried in a topographic half-space of 100 \( \Omega \cdot \text{m} \).](image)
After 7 iterations, the inversion converges, and the data fitting RMS drops to 1.01 (see Figure 5). Figure 3 and 4 shows the comparison between the true model and recovered topographic model. We can see that the inversion results obtained with our inversion code are in good agreement with the true model. The two conductive anomalous bodies are both well recovered from the synthetic data. The position, shape, dipping direction and the angle of the conductive plate are all recovered. Especially, the top of the dipping plate is clearly defined. With increasing depth, the shape of the abnormal body gradually becomes fuzzy. This is consistent with the characteristic of EM diffusion that the high-frequency EM fields of high resolution attenuate with depth when they propagate in a conductive ground, while the low-frequency EM fields have slow attenuation but low resolution. The inversion results also recover the conductive cube under the hill. This result proves that our 3-D inversion code is reliable.

**Figure 2** Transmitting current

**Figure 3** The cross sections of the true model and recovered model.

**Figure 4** Perspective view of the true model and recovered model.
In this paper, we have successfully developed a 3D inversion algorithm for time-domain airborne EM data based on unstructured finite-element algorithm and Gauss-Newton method. By inverting to the synthetic data, we verify its reliability. Our 3D topographic inversion results can well reflect the underground electrical distribution, which lays a good foundation for the subsequent geological interpretation.

Acknowledgements

This work was supported by Natural Science Foundation of China (41830101, 42004118, 42074168, 41704108) and China Postdoctoral Science Foundation (2018M643553).

References


*Figure 5 Inversion parameters versus iterations.*