Utilizing seismic diffractions for imaging and inversion is extremely promising. They exhibit excellent illumination and carry information on the smallest detectable structures in the subsurface. Point diffractor serves as a model for small-scale inclusions, cavities, or structural tips. Edge diffractions originate at faults, highly curved folds, fractures, and pinch-outs. In contrast to diffraction imaging (e.g., Moser and Howard, 2008; Keydar and Landa, 2019), which suppresses the specular component during velocity model based depth imaging, we aim at time-domain diffraction identification free of any a priori knowledge of the subsurface model (Dell and Gajewski, 2011; Rad et al., 2018). It turns out that inspecting properties of specific second order traveltime derivatives (wavefront attributes) allows an unambiguous determination of reflections, point diffractions, and edge diffractions (see Znak et al., 2019).

Theory

Considering the two-way traveltime on the acquisition plane \(T(x^{(1)}; x^{(2)}) = T(x^{(1)}_1; x^{(2)}_1; x^{(1)}_2; x^{(2)}_2)\), we introduce a matrix of the following mixed derivatives at source \(x_s\) and receiver \(x_r\) locations:

\[
\frac{\partial^2 T}{\partial x_i^{(s)} \partial x_j^{(r)}} = \frac{\partial^2 T}{\partial x_i^{(1)} \partial x_j^{(2)}}(x_s; x_r), \quad i, j = 1, 2.
\]

This matrix will play an essential role in our approach for diffraction identification. With no assumptions on the scattering object, it is a general rank 2 non-symmetric \(2 \times 2\) matrix. However, let us examine its properties assuming point and edge diffractors.

For a point diffractor placed at \(x_d\), the traveltime can be decomposed into two direct wave terms:

\[
T^{p.d.}(x_1; x_2) = \tau(x_d; x_1) + \tau(x_d; x_2).
\]

Differentiating confirms that the slowness vector components of one branch are independent of another branch position:

\[
\frac{\partial T^{p.d.}}{\partial x_j^{(2)}}(x_1; x_2) = \frac{\partial \tau}{\partial x_j^{(2)}}(x_d; x_2) \neq f(x_1), \quad j = 1, 2.
\]

Therefore, second differentiating reveals vanishing of all the elements constituting the identification matrix (1) in the case of point diffraction:

\[
\frac{\partial^2 T^{p.d.}}{\partial x_i^{(s)} \partial x_j^{(r)}} = 0, \quad i, j = 1, 2.
\]

Now, let us consider edge diffractions. In the first place, we refer to the geometrical law of edge diffraction (Keller, 1962). It states that an incident ray generates diffracted rays in a cone formed around the edge (see Figure 1). The corresponding opening angle equals the angle between the incident ray and the edge. Analogous formulation is also available for generally anisotropic media in terms of the tangent component of the slowness vector (Rosenbaum, 1967a,b). In fact, we should consider a double cone for
non-zero offsets, as depicted in Figure 1. All rays from the receiver side cone share the same angle to the edge. Hence, according to the edge diffraction law, they all would excite the same unique cone of rays at the source side when reversed in time. These two surfaces cross the acquisition plane resulting in two “focusing curves” – one at the source side and one at the receiver side. Rays propagated back into the subsurface from the focusing curves focus at one particular point on the edge. In the zero-offset case the double cone reduces to the plane orthogonal to the edge (single cone for anisotropy) and the two focusing curves coincide (Znak et al., 2019). Parameterizing the focusing curves \( x^{(1)}(s_1) \) and \( x^{(2)}(s_2) \) with their own natural parameters \( x^{(1)}(0) = x_s, x^{(2)}(0) = x_r \), we can consider the two-way travelt ime \( T^{e.d.}(x^{(1)}(s_1), x^{(2)}(s_2)) \) within the double cone together with the horizontal components of corresponding slowness vector. The key is to understand that such components computed at one side of the cone are independent of the branch position at another side of the cone:

\[
\frac{\partial T^{e.d.}}{\partial x^{(2)}_j} (x^{(1)}(s_1); x^{(2)}(s_2)) \neq f(s_1), \quad j = 1, 2.
\]  

(5)

Indeed, moving the source along the focusing curve does not alter the angle between the incident ray and the edge. Thus, the rays triggered at the receiver side remain unchanged together with their slowness vector components. Differentiating (5) with respect to \( s_1 \) and setting \( s_1 = s_2 = 0 \) yields a linear algebraic system for a tangent to the focusing curve at the source side \( e^{(s)}_j = \frac{dx^{(1)}_j}{ds_1} (0) \):

\[
\frac{\partial^2 T^{e.d.}}{\partial x^{(s)}_i \partial x^{(r)}_j} (e^{(s)}_j)_i = 0, \quad j = 1, 2.
\]  

(6)

In our reasoning, the source and receiver locations can be interchanged resulting in another system for a tangent to the focusing curve at the receiver side \( e^{(r)}_j = \frac{dx^{(2)}_j}{ds_2} (0) \):

\[
\frac{\partial^2 T^{e.d.}}{\partial x^{(s)}_i \partial x^{(r)}_j} (e^{(r)}_j)_i = 0, \quad j = 1, 2.
\]  

(7)

First of all, these relations impose linear dependency on the rows and columns of the identification matrix reducing its rank to 1. Thus, we conclude that the determinant of the identification matrix vanishes in the case of edge diffractions. It trivially vanishes for point diffractions as well. However, inspecting the matrix elements allows an unambiguous discrimination. The matrices in systems (6) and (7) are mutually transposed. The identification matrix becomes symmetric in the zero-offset case and the systems (6) and (7) coincide determining tangents to the single focusing curves. As was demonstrated by Znak et al. (2019), solving an analoguous system with the data-derived wavefront attributes allows for evaluating the focusing curves and for grouping receivers related to one focus point in depth. Extension of this finding to finite offsets seems to be of high importance for diffraction imaging and inversion.

In addition to the representation by source \( x_s \) and receiver \( x_r \) vectors, seismic traces can be referred in terms of the midpoint position \( x = \frac{x_s + x_r}{2} \) and the half-offset \( h = \frac{x_s - x_r}{2} \). It classically serves for numerous seismic processing techniques. Naturally, the second order travelt ime derivatives of both representations can be expressed through each other. Having no space for derivations, we introduce a relation for the identification matrix:

\[
\frac{\partial^2 T}{\partial x^{(s)}_i \partial x^{(r)}_j} = \frac{1}{4} \left( \frac{\partial^2 t}{\partial x_i \partial x_j} - \frac{\partial^2 t}{\partial h_i \partial h_j} + \frac{\partial^2 t}{\partial x_i \partial h_j} - \frac{\partial^2 t}{\partial x_j \partial h_i} \right), \quad i, j = 1, 2.
\]  

(8)

The right side of the equation (8) is composed of finite-offset wavefront attributes, which can be determined using a 3D extension of the finite-offset Common-Reflection-Surface stack (Zhang et al., 2001). In general, it has a symmetric and an antisymmetric part. In the zero-offset case, the antisymmetric part vanishes due to the travelt ime reciprocity leading to \( M^{(s)} - M^{(h)} \) (Znak et al., 2019). Otherwise, the antisymmetric part may significantly contribute. However, for point diffractions, it also vanishes due to the zero matrix on the left-hand side leading to the following equalities:

\[
\frac{\partial^2 t}{\partial x_i \partial h_j} = \frac{\partial^2 t}{\partial h_i \partial h_j}, \quad i, j = 1, 2, \quad \frac{\partial^2 t}{\partial x_1 \partial h_2} = \frac{\partial^2 t}{\partial x_2 \partial h_1}.
\]  

(9)

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For edge diffractions, the identification matrix (8) degenerates: \( \text{det} (M^{(x)} - M^{(h)} + M^{(x,h)} - M^{(h,x)}) = 0. \)

**Numerical tests**

In order to verify the criterion of edge diffraction identification, we simulate the wavefront attributes for a model with 3-D velocity gradient (Figure 2) and for a homogeneous VTI model (Figure 3). For both models, we applied the Fermat’s principle estimating the two-way traveltime combined with the finite differences to approximate the second order derivatives. In Figures 4 and 5, we illustrate the validity of the finite-offset criterion with the augmented identification matrix. The quantity of the zero-offset criterion, where the antisymmetric part is missing, strongly deviates for the offsets under consideration.

**Conclusions**

The finite-offset wavefront attributes enable the time-domain pre-stack recognition of diffractions and the determination of their type. The receiver selection technique leading to edge diffraction focusing is
carried for the case of arbitrary finite offsets.

**Figure 5** Determinant of the identification matrix (zero- and finite-offset versions) for different midpoint locations. The model is anisotropic (Figure 3). Top: \( h = (400, 0) \) m. Bottom: \( h = (0, 400) \) m.

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**References**


