Introduction

CO₂ geo-sequestration is an emission reduction strategy that allows captured CO₂ mainly from fossil fuel energy be sequestered in geological formation. CO₂ injected into geological deep saline formation is in supercritical (scCO₂) phase at depth below 800m where the temperature and pressure are above the critical point of CO₂. The flow and transport of the injected and resident fluid (brine) are governed by pressure gradient, aquifer flow and buoyancy. As a result, the plume of scCO₂ rises and spreads underneath the caprock depending on the caprock morphology (Ahmadinia et al. 2019). The lateral migration of scCO₂ at the caprock poses a risk of leakage if there exist escape pathways such as fractures, faults, or abandoned wells. The dissolution of scCO₂ in brine is preferred because brine saturated with CO₂ is less mobile and dissolved CO₂ is transported to the bottom of the aquifer reducing the chance of leakage. At an early stage, diffusion is the mechanism that transports dissolved CO₂ from the layer of CO₂ saturated brine towards the bottom of the aquifer. The diffusing CO₂ provides opportunity for further dissolution. The diffusive boundary layer of brine saturated with CO₂, which is heavier than the underlying brine, continues to grow. The system becomes unstable when the boundary layer reaches a tipping point to set off density driven convection. The convective process accelerates the removal of CO₂ from the supercritical phase compared to pure diffusion and is therefore beneficial to the underground CO₂ storage security.

Density-driven convection in a clear fluid layer (Rayleigh-Bernard convection) and in a porous medium (Horton-Rogers-Lapwood convection) due to a constant temperature gradient has been studied. Lindeberg and Wesselberg (1997) applied the procedure in Horton-Rogers-Lapwood convection for the case relevant to CO₂ sequestration. Their result offered insight, for example, that geothermal convection could be neglected when a CO₂ concentration gradient exists due to the difference in thermal and molecular diffusivity. Subsequently, many other studies on density driven convection in homogeneous and heterogeneous systems have been reported (Luther et al. 2019, Green and Ennis-King 2014, Farajzadeh 2010).

In many studies on density driven convection, heterogeneous systems are commonly described based on variable permeability field which could be generated using Spectral Fourier method and sequential Gaussian Simulation method (Farajzadeh 2010). Dispersed barriers representing shale within natural sedimentary formation is another method for modelling a heterogeneous permeability distribution (Green and Ennis-King 2014). Similarly, the onset of convection with a simple permeability function varying exponentially in the vertical direction (Ryoo and Kim 2018) has been reported. Sedimentary formations (aquifer formations) exist naturally in layers but the onset of convective instability in a layered porous medium has not received significant attention. The onset of convective instability in a layered system have been studied with a steady base profile (Mckkbin and O'Sullivan 1981). However, research on the onset of convective instability in a layered porous medium with a transient base profile is scarce. Permeability variation in a layered form affects the growth of perturbations and the long-term flow of brine. We investigate the effect of permeability heterogeneity relevant to a layered porous medium on the onset of convective instability. Linear stability analysis (LSA) incorporating the quasi-steady state approximation (QSSA) is used to analyse the stability of the system due to the time dependency of the base state profile. The onset time is defined as when the introduced perturbations just begin to grow. The location of the peak of the concentration perturbation at the onset time is used as a reference point of the convective instability.

Model and Governing Equations

We model the onset of density driven convective instability in a porous medium with an embedded low-permeability layer. We define a three-layer system where each layer is a homogeneous and isotropic brine-saturated porous medium, but the entire system is heterogeneous in permeability (Figure 1a).
Figure 1: Shows (a) a porous medium with a low-permeability layer (b) the piecewise permeability function describing the permeability in the system in dimensionless units.

Each of the three layers has a depth \(a, b-a, \text{ and } H-b\), from the top to bottom respectively, so that the system has a thickness of \(H\). The outer top and bottom impermeable boundaries are maintained at constant \(\text{CO}_2\) concentration in the fluid and no-flux of \(\text{CO}_2\) concentration respectively as used in previous studies (Raad and Hassanzadeh 2016). At the internal boundaries, we require the pressure to be continuous. We commence this investigation with no \(\text{CO}_2\) concentration within the domain of interest (figure 1a) and limit the analysis to a single-phase flow of brine with dissolved \(\text{CO}_2\) transport. We adopt the Cartesian coordinate system with the origin located at the top left of the composite system and restrict our analysis to 2-dimensions. The system of equations is

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial z} = 0
\]

\[
u \frac{1}{k(z)} = \frac{1}{\mu} \frac{\partial p}{\partial x}
\]

\[
u \frac{1}{k(z)} = -\frac{1}{\mu} \left( \frac{\partial p}{\partial z} + (1 + \beta \Delta c) \rho u g \right)
\]

\[
\phi \frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial z} = \phi D \left( \frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial z^2} \right)
\]

where \(k\) is the variable permeability, \(u\) and \(v\) are the Darcy velocity components in the horizontal and vertical direction respectively, \(c\) is \(\text{CO}_2\) dissolved concentration, \(g\) is gravity, \(D\) is the diffusivity of \(\text{CO}_2\) in brine saturated porous medium, \(p\) is the pressure, \(\mu\) is the viscosity and \(\phi\) is the porosity. The density is assumed to be linearly dependent on the concentration and Boussinesq approximation is made. The governing equations are made dimensionless using the variables: \(x' = \frac{x}{H}, z' = \frac{z}{H}, a_H = \frac{a}{H}, b_H = \frac{b-a}{H}, c' = \frac{c}{c_s}, \mu' = \frac{\mu u}{H^2}, v' = \frac{v}{H^2}, t' = \frac{t}{H^2}, P' = \frac{\rho \mu D}{\phi \mu D}, R_a = \frac{k_s \Delta p H}{\phi \mu D}\).

\(\Delta p = \rho_0 \beta C_s, k' = \frac{k}{k_s}\) where \(C_s\) and \(k_s\) are the solubility of \(\text{CO}_2\) in brine and permeability at the top layer respectively. The governing equations are linearized and then analyzed using LSA based on QSSA to handle the transient base profile. Also, the piecewise permeability distribution in the layered system is approximated with a continuous function based on a finite difference implementation of the linearized equation near the internal boundaries.

**Results**

The relationship between Rayleigh number and onset time in both homogeneous and heterogeneous systems are presented in figure 2. We obtained the common relation \(t = a R_a^{-\frac{1}{2}}\) where our prefactor \((a \approx \ldots\)}
57.12) is similar to prefactors in previous work $a \approx 56$ (Raad and Hassanzadeh 2016) for the homogeneous system $k' = 1$.

Figure 2: Dimensionless onset time of convective instability versus Rayleigh number for homogeneous and heterogeneous systems.

The heterogeneous models are described with the Rayleigh number of the global homogeneous porous medium ($Ra$) and the location of the low-permeability layer ($a_H$). To obtain insight into the effect of the low-permeability layer on the onset of convective instability, we consider cases where the thickness of the low-permeability layer is constant ($b_H - a_H = 0.2$) and $a_H$ is varied. Physically, the low-permeability layer is close to the bottom boundary when $a_H$ is large. The investigation is restricted to the cases where the low-permeability layer is an order of magnitude less than the permeability of the global porous medium (figure 1b). Figure 2 shows that the onset of convection is affected due to the presence of the low-permeability layer at low Rayleigh number ($Ra \leq 200$). At a significantly high Rayleigh number, the onset time in the homogeneous system and in the heterogeneous systems are comparable.

Figure 3: The location of the peak of the concentration eigenfunction within the system in (a) homogeneous case (b) heterogeneous case ($a_H = 0.1$) (c) heterogeneous case ($a_H = 0.4$) (d) heterogeneous case ($a_H = 0.7$).
The profile of $a_{th} = 0.1$ shows that the onset of convection is delayed but becomes early in comparison to homogeneous system as Rayleigh number increases ($50 \leq Ra \leq 200$). At $Ra = 50$, the onset time has a non-monotonic effect as $a_{th}$ increases from 0.1 to 0.7. This behaviour of the onset time is due to the location of the peak of the concentration perturbation relative to the location of the low-permeability layer.

Figure 3 shows the location of the peak of the concentration perturbation represented by the peak of the concentration eigenfunction in the homogeneous and heterogeneous systems. In the homogeneous system (Figure 3a), the peak of the concentration perturbation is near the top of the boundary for large Rayleigh number, indicating that a high permeability system readily propagates small perturbations earlier in a homogeneous system. The onset of convection is delayed when the peak of the concentration perturbation is within or below the low-permeability layer in a heterogeneous system in comparison to the homogeneous case. However, the onset of convection is early when the peak of the concentration perturbation is near the top of low-permeability layer. The early onset of convective instability near the low-permeability layer is due to the additional disturbances originating from the point where there is a change in permeability. When the peak of the concentration eigenfunction is well above the low-permeability layer, the effect of the low-permeability layer is less significant.

**Conclusions**

Formation heterogeneity is commonly based on random and regular variation of permeability. However, the effect of a layered permeability heterogeneity on the onset of transient convective instability has not received much attention. We employed LSA with QSSA in a finite difference framework and approximated the piecewise permeability with a continuous function. Results show that the onset of convective instability depends non-monotonically on the presence of a low-permeability layer. In the heterogeneous systems, the onset of the convective instability is early when the peak of the concentration eigenfunction is above and near the low-permeability layer in comparison to homogeneous system. This onset is delayed when the eigenfunction peak is within or below the low-permeability layer.

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**References**


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