An Empirical Method for the Optimal Setting of the Gravimetric Inverse Problem

Introduction

The use of the Earth’s gravity field data for geophysical exploration purposes is nowadays quite common because this technique consents to retrieve a geological knowledge over extended areas with high spatial resolutions. Moreover, it can give complementary information where other invasive or expensive techniques, such as seismic acquisitions, fail (e.g. in the recovery of geometries of main geological horizons beneath a thick salt layer). Recent dedicated satellite gravity missions, such GRACE and GOCE, together with the exploitation of offshore satellite altimetry, have paved the way to the realization of a variety of new global gravity field models (GGMs), characterized by spatial resolutions of about 3 – 4 km (in terms of gravity anomaly) at ground level and high-accuracy, i.e. about 5 mGal globally and 3 mGal offshore (Zingerle et al., 2020). This kind of combined models are a valuable source of information to study the geological evolution and characterization of the lithosphere structure, especially at regional scale.

In the present work we will discuss some preliminary technical aspects related to a proper use of these global gravity models to perform a full 3D inversion. In particular, we will empirically address the questions whether the classical planar approximation is sufficient for regional inversions or if a spherical approximation is required and what is the best functional, i.e. classical gravity anomalies ($\delta g$) or second radial derivative of the anomalous potential ($T_{rr}$), to perform such an inversion. The answers to these questions, together with the optimal sizing of the 3D volume area to be modeled, given a specific target, constitute fundamental problems to be tackled before the gravity inversion is carried out. In fact, if not properly solved, they can introduce unnecessary computational work (e.g. if spherical approximation is used instead of a planar one or if the considered volume is discretized into a set of too small elements) or even worse they can be source of errors in the final results (if viceversa the planar approximation is used when a spherical one is required or if the volume is not correctly dimensioned). In the following we will suppose that our inverse problem consists in estimating the densities of a set of volumetric elements (voxels) used to model our 3D volume, i.e. a problem similar to the classical inversion presented in Li and Oldenburg (1998). However most of the reasoning can be extended to each kind of inversion.

Method and Theory

The setting and proper sizing of a gravity inverse problem is a complex task, which is usually underestimated, since nowadays the increasing computational power allows to perform very complex gravity inversions. However, if properly considered, it can reduce the computational time required in order to obtain a solution or even avoid to introduce modeling errors in the inversion itself. The proposed approach follows the subsequent steps:

1. selection of the most appropriate functional ($\delta g$ vs $T_{rr}$);
2. selection of the approximation (planar vs spherical);
3. selection of the 3D model spatial resolution;
4. selection of the maximum depth of the 3D model to be included within the inversion;
5. selection of the size of the border region.

Points 3. to 5. have been already discussed in Sampietro and Capponi (2019) and we will report in here, for the sake of completeness, only the numerical results obtained in the considered case study. As for points 1. and 2., the simplest way to face these problems consists in exploiting forward operators on a given a-priori model of the studied volume.

Starting from the discussion of the choice of the best functional, we follow the reasoning presented in Pappa et al. (2019) to study the sensitivity of GOCE data to density variations at different depths. We will suppose to have a realistic model of the 3D volume area to be studied, divided in different layers of given thickness. Given this model and by exploiting the forward operator, we can compute the relative
standard deviation of the signal (due to each depth) with respect to the whole forward of the reference model. In this way it is possible to compare, for the considered functionals, the relative amount of signal coming from each specific depth. At this point, given the average depth of the target density discontinuity studied, the most appropriate functional can be selected as the one with the largest relative signal. Once the functional has been selected we can move to the selection of the most appropriate approximation (i.e., planar vs. spherical). As it is well known, we recall that when the inversion is performed on a regular grid in planar approximation, FFT algorithms can be successfully exploited thus notably reducing the computation time. However, when large areas are considered, this approximation can be too rough, introducing fictitious unwanted signals. The solution of this problem can be tackled by performing the forward of the initial model in planar approximation (by means of right rectangular prism voxels) and in spherical approximation (by means of tesseroidal voxels) and then comparing the resultant signals. When comparing the two signals it has to be kept in mind that, with classical inversion, the average density of the layers cannot be estimated, but we can retrieve only the density contrast with respect to an unknown background (Reguzzoni et al., 2020). In fact, the inversion is in general performed starting from a functional of the anomalous potential and this implies the removal, from the actual observed gravity, of the effect of the so-called normal signal (i.e., the gravitational effect of an ellipsoidal Earth, with mass equal to the actual total Earth mass, and internal unknown density distribution with ellipsoidal symmetry). When moving to the planar approximation this ellipsoidal shaped unknown density distribution is reflected into a set of Bouguer plates, while, when moving to the spherical approximation, it is reflected into a set of spherical shells with constant densities. As a consequence when comparing two gravimetric signals the average difference is negligible and only the standard deviation (std) should be considered. So, to choose the preferred approximation, if the difference (in terms of standard deviation) between the two signals is smaller than the observation error than the planar approximation can be used, otherwise we have to move to the spherical one. Note that while for the right rectangular prism there is a well-known solution (see Nagy (1966)), for the tesseroid no closed analytical formula are available and one has to use approximated formulas (e.g., see Asgharzadeh et al. (2007)).

Finally, once even the proper approximation has been selected we have to properly dimension the volume, i.e., define the size of the volumetric elements used to discretize the 3D model, the maximum depth to which the volume should be modeled and the size of the border required to avoid boundary effects. For all these purposes, the method described in Sampietro and Capponi (2019) can be followed, thus obtaining a procedure for the proper setting of a general gravity inversion problem.

Example

In the current section, the proposed procedure has been applied to the Central-Eastern Mediterranean area in order to set all the elements required to perform the 3D inversion. The study area extends from 16.5° E to 36.5° E in longitude and from 31° N to 37° N in latitude for a total of about 2000 km × 600 km. As described in the previous section the first preliminary operation consists in building a realistic 3D volume of the considered area. To this aim we created a simple model till a depth of 52 km, composed by the following layers: bathymetry with the top of the model fixed at zero level, Plio-Quaternary sediments from Capponi et al. (2020); Messinian sediments from Haq et al. (2020); basement computed by merging (with a kriging interpolation) data and models from previous studies (Makris and Yegorova, 2006; Sampietro et al., 2018); Moho depth from Parker inversion. For such a regional study we will suppose as main target the recovery of the Moho and basement depths while we will use as observations data from the XGM2019e model (Zingerle et al., 2020), supposing an observation error of 3 mGal (std). As for the densities, we will suppose 2400 kg/m³ for the Pre-Messinian sediments, 2160 kg/m³ for the Messinian layer, 2400 kg/m³ for the Plio-Quaternary sediments, 2800 kg/m³ for the crust. A linear gradient of 11 kg/m³/km has been considered for the Pre-Messinian sediments and for the crust. The upper mantle density model as been taken from Blom et al. (2020).

Starting from the choice of the functional, in Figure 1 we report the relative amplitude of the two gravity signals as a function of depth. As expected, the largest amount of signal, in both cases, is found in correspondence of the average depth of the main discontinuity surfaces (dashed line in the figure). It can also be observed that, up to a depth of about 10-15 km, $T_r$ (red line) presents a relative amplitude larger than the one of $\delta g$, meaning that in this range $T_r$ is more sensible to density variations. On the contrary for density variations below 15 km, the gravity anomaly seems to perform better. For this reason, for our example in which the targets of the inversion are the depth to basement and the Moho depth,
Figure 1 Relative amplitude of the gravitational field effect in terms of gravity anomalies (blue lines) and second radial derivative (red line) as a function of depth. Dashed line represents the average depth of the main discontinuities surfaces.

Figure 2 Gravitational effect in terms of gravity anomalies in planar approximation and difference between spherical and planar approximation.

$\delta g$ should be the optimal functional to be used. Moving to the choice of the approximation, in Figure 2 the gravitational signal of the 3D volume in planar approximation is shown together with the difference with respect to the spherical one. As it can be seen, even if the difference shows features correlated to the signal itself, its standard deviation is quite small (0.97 mGal) and as a consequence can be considered negligible if compared to the assumed observation error of 3 mGal (std). This means that for this example the planar approximation is sufficient. Note that for this test, in which the a-priori model is made by approximatively $600 \times 200 \times 100$ voxels (in the $x$, $y$ and $z$ direction respectively), the forward computation takes few seconds in planar approximation and about 30 minutes with the spherical one, thus confirming the importance of the choice of the proper approximation from the computational point of view.

Considering the setting of the spatial resolutions in the $x$ and $y$ directions, the method proposed in Sampietro and Capponi (2019) has been applied. Results are shown in Figure 3 where the gravitational effect of the model at different spatial resolution has been compared with the one obtained with a resolution in the $x$ and $y$ directions of 100 m. It can be observed that the downsampling of the initial model at a resolution of about 4 km is appropriate, since it entails an error smaller than 3 mGal. As for point 4. and 5. namely the selection of the maximum depth to be modeled in the inversion and the size of the borders, applying the same methodology described in Sampietro and Capponi (2019), we found that, since we are modelling till a depth of 52 km, to avoid signals from the deeper mantle, the gravitational effect of the portion of mantle from 52 km till a depth of at least 200 km should be removed from the observation signal. Moreover, a border of at least 300 km is required.
Conclusions

In the current work we presented a procedure to proper set a gravimetric inversion problem, given the accuracy of the available gravity observations and the depth of the main target to be investigated. The proposed approach consists in building a realistic a-priori model of the main density variations in the area to be investigated and in exploiting the forward operator, performed with different approximations and resolutions. The proposed method has been successfully applied to set a 3D gravity inversion in the Central-Eastern Mediterranean area. The example shows that, even if a quite large area is considered, the planar approximation is sufficient, given an observation error of 3 mGal (std). Moreover if the target of the study are deep discontinuities (below 15 km), the use of classical gravity anomalies is preferable, otherwise second radial derivative of the gravitational potential are more appropriate. In order to perform the inversion the volume should be discretized in voxels of size in the $x$ and $y$ directions of 4 km, it should be framed with a border of 300 km and the effect of the densities anomalies at least up to 200 km should be removed from the observations.

References