Introduction

Ray theory and wave equation are two main types of seismic wave numerical simulation and imaging methods. Among them, ray theory describes the seismic wave propagation with high frequency approximation, which has the advantages of high efficiency, flexible adaptation to irregular observation data and surface fluctuation. However, the imprecision of its wave field description is also caused. The wave field simulation method based on one-way wave equation can not deal with the large angle propagation of wave field and the propagation of diving wave. The combination of the two method is the development direction, which balances the efficiency and accuracy.

Brandsburg-Dahl and Etgen (2003) proposed a method combining ray tracing with one-way wave equation. However, the method only uses the traditional one-way wave equation to describe the wave propagation in the ray beam. Sava and Fomel (2005) proposed wave extrapolation in Riemannian coordinates. Shan (2003,2005,2008) used the anisotropic one-way wave equation to image the steep dip angle in the inclined coordinate system, but this method could not process the diving wave. Cheng (2012) studied the one-way wave migration in the ray center coordinate system of isotropic medium, and simulated the local wave field with higher accuracy and the ability to process the diving wave.

The migration method based on ray theory and wave equation have their own advantages and disadvantages. Therefore, we develop the wave equation and its one-way wave extrapolation equation in the ray center coordinate system in VTI medium, so as to use the equation to realize the propagation of local wave field in the beam of the ray. Compared with the traditional one-way wave equation by Brandsburg-Dahl and Etgen (2003), this method can describe the local wave field more accurately. Compared with Sava and Fomel (2005) in Riemannian coordinate system, local characteristics of wave field can be considered more when wave field is propagated in local beam of the ray. This method not only combines the flexibility of the ray theory method, but also can better describe the wave field accurately in the local beam of the ray, so as to solve the migration imaging problem of complex structures in the anisotropic medium and tomography of anisotropic parameter inversion and estimation. Numerical experiments prove the correctness of our proposed method.

The transformation between the ray center coordinate and Cartesian coordinate

In three-dimensional case, the ray center coordinate system can be represented by figure 1, where \( \Omega \) is the ray center, \( R \) is a point on the center ray, \( r \) represents the ray direction vector, \( \Sigma^\perp \) is the plane perpendicular to the center ray for point \( R \). \( \mathbf{e}_1 \) and \( \mathbf{e}_2 \) respectively represent two vertical unit vectors in the plane \( \Sigma^\perp \) which is perpendicular to central ray, with the intersection point \( R \) of the center ray \( \Omega \) as the origin point. These two vectors and the center ray form the center coordinate system of the ray, and \( q_1,q_2,q_3 \) are the coordinate value of the three coordinate directions. The point \( R' \) in the figure 1 represents a point near the center ray.

![Figure 1 Three-dimensional ray center coordinate system and its properties. \( R \) is on the center ray, and the coordinate value of \( R \) in the ray center coordinate system is \( q_1 = q_2 = 0, q_3 = s, R = (0,0,s) \).](image)
For point $R$ near the center ray, its coordinate $q_3$ is the same as point $R$, so let the coordinate $q_3$ of any point in the plane $\Sigma^-$ which is perpendicular to the center ray be $q_3 = s$.

The expression of the radial vector of point R in the ray center coordinate system is:

$$
\mathbf{r}(q_1, q_2, s) = \mathbf{r}(0, 0, s) + q_1 \mathbf{e}_1(s) + q_2 \mathbf{e}_2(s)
$$

$$
d\mathbf{r} = \frac{dr}{ds}ds + \frac{dr}{dq_1}dq_1 + \frac{dr}{dq_2}dq_2 = \left[ \frac{dr(0, 0, s)}{ds} + q_1 \mathbf{e}_1 + q_2 \mathbf{e}_2 \right] ds + \mathbf{e}_1 dq_1 + \mathbf{e}_2 dq_2
$$

In anisotropic medium, the ray path and wavefront diffusion direction do not coincide because the group velocity and phase velocity are not equal. There is an angle between the direction of the wave vector and the direction of the ray path. Assume the angle to be $\gamma$. Then formula (2) can be expressed as:

$$
d\mathbf{r} = \left[ \mathbf{e} - q_1 (\mathbf{p} \cdot \mathbf{p})^{-1} \left( \mathbf{e} \cdot \frac{d\mathbf{p}}{ds} \right) \mathbf{p} - q_2 (\mathbf{p} \cdot \mathbf{p})^{-1} \left( \mathbf{e} \cdot \frac{d\mathbf{p}}{ds} \right) \mathbf{p} \right] ds + \mathbf{e}_1 dq_1 + \mathbf{e}_2 dq_2
$$

$$
= \left[ \cos \gamma t - q_1 |\mathbf{p}|^{-1} \left( \mathbf{e} \cdot \frac{d\mathbf{p}}{ds} \right) \mathbf{p} - q_2 |\mathbf{p}|^{-1} \left( \mathbf{e} \cdot \frac{d\mathbf{p}}{ds} \right) \mathbf{p} \right] t ds + \mathbf{e}_1 dq_1 + \mathbf{e}_2 dq_2
$$

$$
= h t ds + \mathbf{e}_1 dq_1 + \mathbf{e}_2 dq_2
$$

where $h = \cos \gamma - q_1 |\mathbf{p}|^{-1} \left( \mathbf{e} \cdot \frac{d\mathbf{p}}{ds} \right) - q_2 |\mathbf{p}|^{-1} \left( \mathbf{e} \cdot \frac{d\mathbf{p}}{ds} \right)$. Formula (3) establishes the connection between the ray center coordinate system and the Cartesian coordinate system, and the transformation between the two coordinate systems can be realized by constructing the transformation matrix of formula (3).

**One-way wave equation in ray center coordinate system**

The dispersion formula of q-P wave equation in VTI medium is:

$$
k_s^2 = \frac{v^2}{v_r^2} \left( \frac{\omega^2}{v_r^2} - \frac{\omega^2 (k_x^2 + k_y^2)}{\omega^2 - 2v^2 \eta (k_x^2 + k_y^2)} \right)
$$

where $v_r$ is the NMO velocity $V_{\text{NMO}}$; $v_r$ is the vertical velocity $V_{p_0}$. They satisfy the following relationship:

$$
V_{\text{NMO}}(0) = V_{p_0} \sqrt{1 + 2\delta} = \frac{\varepsilon - \delta}{1 + 2\delta}
$$

According to the coordinate relationship discussed above, the dispersion formula in Cartesian coordinate system can be transformed into the ray center coordinate system, and the dispersion formula in two-dimensional ray center coordinate system is obtained as:

$$
\left( v_r \frac{n_r^2}{h^2} + v^2 \frac{n_x^2}{h^2} \right) k_x^2 + \left( v_r^2 t_x^2 + v^2 t_z^2 \right) s^2 - \frac{2}{h} \left( v_r t_n \frac{n_r}{h} + v^2 t_n \frac{n_z}{h} \right) k_x k_s
$$

$$
-i \left( v_r \frac{n_r}{h} \frac{\partial}{\partial s} \left( \frac{n_r}{h} \right) - v^2 \frac{\partial}{\partial q} \left( \frac{1}{h} \right) \right) + v^2 \left( \frac{n_r}{h} \frac{\partial}{\partial s} \left( \frac{n_r}{h} \right) - v^2 \frac{\partial}{\partial q} \left( \frac{1}{h} \right) \right) k_x
$$

$$
+ i \left( v_r \frac{n_r}{h} \frac{\partial}{\partial s} (t_n) + v^2 \frac{\partial}{\partial q} (t_z) \right) k_s = \omega^2
$$

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where \( \mathbf{t} = (t_1, t_2) \) is the basis vector in ray direction; \( \mathbf{n} = (n_1, n_2) \) is the basis vector perpendicular to the direction of the ray.

In order to get one-way wave equation in the ray center coordinate, we need to solve for the wave number \( k_s \) as

\[
k_s = \frac{ic_{ss} - c_{sq} k_q}{2} \pm \sqrt{\alpha \omega^2 \left( \frac{c_s}{2c_{ss}} \right)^2 + \left( \frac{c_{sq}}{c_{ss}} \right)^2 - \frac{c_{q}^2}{c_{ss}}} \frac{k_q}{k_q^2} + \left( \frac{ic_q}{c_{ss}} \frac{ic_{sq}}{2c_{ss}} \right) \left( \frac{k_q}{k_q^2} \right)^2
\]

where, the positive sign is the extrapolation along the negative direction; The minus sign is the extrapolation along the forward direction. After derivation and inverse Fourier transform, the one-way wave equation in the frequency space domain is obtained as:

\[
\frac{\partial u}{\partial s} = i \left( \frac{ic_s}{2c_{ss}} - k_0 \right) u - \left[ \frac{c_{sq}}{2c_{ss}} + \frac{1}{2k_0} \left( \frac{ic_q}{c_{ss}} - \frac{ic_{sq}}{2c_{ss}} \right) \right] \frac{\partial u}{\partial q} + \frac{i}{2k_0} \left[ \frac{c_{sq}}{2c_{ss}} - \frac{ic_{sq}}{2c_{ss}} \right] \frac{\partial^2 u}{\partial q^2}
\]

where

\[
c_{ss} = v_s^2 \left( \frac{n_1}{h} \right)^2 + v_t^2 \left( \frac{n_2}{h} \right)^2 \quad c_{sq} = -2 \left( \frac{v_s^2}{h} \frac{n_1}{h} \frac{t_1}{h} + v_t^2 \frac{n_2}{h} t_2 \right) \quad c_q = -v_s^2 \frac{n_1}{h} \frac{\partial (t_1)}{h} \frac{\partial (t_1)}{h} + v_t^2 \left( \frac{t_1}{h} \right)^2 \quad c_{ss} = \frac{\alpha^2}{c_{ss}} \left( \frac{c_s}{2c_{ss}} \right)^2
\]

**Numerical tests of simulation and migration**

We first verify the reliability of our forward modeling method. This model is of constant velocity with 3000m/s, \( \varepsilon = \delta = 0.065 \), \( \alpha = \beta = 10m \), \( nx = 401 \), \( nz = 301 \). Figure 2 shows snapshots at 500ms, and snapshots of wave fields at depths of 1500m and 2500m were taken for comparison respectively.

**Figure 2** Comparison of final wave between traditional q-P wave and our method:

(a) Final wave which is formed by one-way wave of our method; (b) final wave of traditional q-P wave; (c) comparision between (a) and (b) at the depth of 1500m (i.e. the upper red line); (d) comparision between (a) and (b) at the depth of 2500m (i.e. the lower red line).

As is shown in figure 2, our forward modeling method can approximate the results of the traditional two-way wave equation well. Then, we compare the migration results between one-way wave equation and RTM of Hess model (figure 3) in figure 4.
Figure 3 Hess model: (a) $v_p$; (b) $\varepsilon$; (c) $\delta$.

Figure 4 Imaging results of prestack depth migration in Hess model: (a) traditional one-way wave migration results; (b) RTM migration results; (c) one-way wave migration results of our method.

From the imaging results in figure 4, we can see that the imaging of traditional one-way wave is the worst. The one-way wave imaging method proposed by us can produce more precise images of thin layer and more accurate imaging results at larger angles of the fault. Compared with RTM imaging method, this method has high flexibility. Wave field simulation can be carried out only by a one-way wave equation. Moreover, there is no high-frequency approximation. The imaging results show that this method is not inferior to RTM imaging.

Conclusions

In this paper, we derive the one-way wave equation in the ray center coordinate system, and compare the spherical wave synthesized by our proposed method with the spherical wave of the traditional two-way wave equation. Numerical experiments show that the proposed method can well approximate the spherical wave of the two-way wave equation. The imaging results of thin layer and fault of large angle are more accurate.

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