Introduction

Reverse Time Migration (RTM) is a well-established method for large-scale imaging of seismic data that uses the two-way wave equation. It can image complex geological structures with vertical dips, multiple arrivals and lateral variations of wave velocity. Very often certain assumptions on a medium response are taken in seismic imaging, one of those being that the substrata are perfectly (or nearly) elastic. Effectively, over the typical seismic bandwidth geological media respond anelastically and absorb part of the propagating seismic energy because of the existence of internal friction. That phenomenon is generally referred to as intrinsic attenuation and it is quantified by the quality factor $Q$ which is a measure of the quantity of energy lost per cycle. Ignoring intrinsic attenuation can potentially result in unfocused seismic images with weakening amplitude in depth. Therefore, considering the effect of intrinsic attenuation is paramount to improve the accuracy of those images.

Considering $Q$ in RTM imaging (QRTM) has been introduced previously. Time-reversing attenuating wave equations has inherent difficulties as it generally leads to unstable solutions due to amplification of numerical errors. That issue has been addressed through different methods. For example, Deng and McMechan (2007) introduced the use of a damped wave equation, reversing the sign of $Q$ along with time-reversing. However, that approach does not account for dispersion, yielding inaccurate quantification of intrinsic attenuation in wave modelling. Alternatively, time-reversing attenuating wave equations can be stabilised by filtering out high-frequency components when using pseudo-spectral methods or by introducing regularized versions of pseudo-differential operators (Zhang, 2010). However, filtering or regularizing computed wavefields has two main drawbacks. Firstly, those methods may not be computationally efficient. Secondly, filtering or forcing regularization over modelled wavefields can potentially remove spectral components from the seismic image which are not necessarily spurious, as those components cannot be isolated in the Fourier domain. Alternatively, reverse modelling can be stabilised by eliminating the attenuation component from the modelling operator while keeping the dispersion term (Zhu, 2014; Li et al., 2016). That method has the key advantage of keeping the correct kinematics of wave propagation while preserving the amplitudes. In other words, separating pure attenuation and dispersion terms yields a wave equation which is unattenuated. Hence, it places peaks and troughs at the correct space and time positions yielding correct interference between forward and propagated wavefields and subsequently an improved seismic image. This approach, while elegant, has some drawbacks. First, it is suitable only for specific models of intrinsic attenuation, e.g. Kjartansson’s model (Kjartansson, 1979), hence excluding a vast array of widely used implementations based on standard linear solids. Second, as it is based on the use of fractional partial derivatives it also limits the type of numerical methods that can be used. Third, dealing with fractional partial derivatives is not straightforward.

Rather than handling directly numerical aspects inherent to time-reversing attenuating wave operators, other alternative approaches have been focused on isolating the effect of intrinsic attenuation in the seismic data. For example, the effect of $Q$ can be filtered out directly from each trace using time-invariant filters designed for a given model of $Q$ (Hargreaves, 1991; Ferber, 2005). Nonetheless, most of those previous approaches are not suitable for handling lateral variations of $Q$, and for that reason they are not rigorous in handling the accumulated effect of attenuation over the propagation path of seismic waves. Fletcher et al. (2012) introduces a filtering approach which can handle the propagation path based on the computation of a compensation filter by estimating the attenuative traveltime. This approach requires propagating a wave equation four times and using an ad hoc parameter for controlling the level of attenuation. A key drawback of compensating intrinsic attenuation based on travel-times is handling multipathing. Hence, that approach may struggle when handling the imaging of strong heterogeneities.

In this paper we introduce an alternative method which also compensates for the effect of attenuation. Our method determines the effect of $Q$ by generating attenuated and non-attenuated synthetic data with the same tomographic velocity model. Then, a filter is computed matching the attenuated into the non-attenuated synthetic data. Finally, that filter is used to compensate the data prior to carrying out conventional RTM (i.e. ignoring attenuation). Our approach is stable and general for any type of medium parameterization, e.g. anisotropy, viscoelasticity, etc. As the filters are computed from data modelled with a wave equation, it accounts for directivity effects and multipathing. In addition, it is independent from ad hoc parameters related to the attenuation model. In the following sections we
outline technical details on our method, as well as synthetic and real data examples which demonstrate its effectiveness.

**Method**

Reverse time migration places reflectors at their correct positions in depth by propagating backwards in time events recorded at a boundary. Generally, RTM forward propagates a source wavefield with boundary-data defined as

\[
\begin{align*}
Lp_s(x, t) &= s(x, t) \\
Ld_{cal} (x, y, t, x_s) &= p_s(x, y, z = 0, t)'
\end{align*}
\]

and back propagates a receiver wavefield with boundary data

\[
\begin{align*}
Lp_r(x, t) &= 0 \\
Lp_r(x, y, z = 0, t) &= d(x, y, x_s, t)'
\end{align*}
\]

The operator \( L \) is a wave operator which depends on the medium parameters (e.g. velocity, anisotropy, and/or attenuation), \( p_s \) is the source wavefield, \( p_r \) is the receiver wavefield, \( x=(x,y,z) \) is the position in space, \( t \) denotes time, \( d_{cal} \) is the wavefield at the boundary for the source wavefield, and \( d \) denotes observed data. The correct position of the reflectors in depth is given by the zero-lag cross correlation between forward- and back-propagated wavefields (Baysal et al., 1983).

The differentiating factor in our approach is how compensating for the effect of intrinsic attenuation in the recorded data is carried out. We can obtain a good estimate of that effect if a suitable model of \( Q \) is known beforehand (this is a necessary condition for any practical viscoacoustic RTM scheme). In addition, we assume implementations for seismic modelling with and without attenuation are available.

Our method utilizes these implementations to generate data with and without anelastic loss. We then can generate a filter mapping the data affected by attenuation into the data not affected by attenuation. That filter contains the effect of attenuation over the path of propagation of the wavefield for each shot. After determining each matching filter, the effect of attenuation in the real data is filtered out. Finally, the data corrected by filtering can be migrated using a migration code which does not need to account for intrinsic attenuation. This approach circumvents any stability issues related to back-propagation with an attenuating equation, as it does not depend on such an operation. As migration velocity models are generally smooth, they do not generate scattering events. These can be inserted by creating a pseudo-density volume from a preliminary migrated image to insert scatterers when simulating the data to generate the compensation filters. This operation is effectively similar to a demigration. In conclusion our algorithm for compensated QRTM is summarised by the following steps:

1. Migrate seismic data without accounting for attenuation
2. Generate records with and without attenuation to generate scattered events, using the migrated image obtained in 1.
3. Estimate filters matching shots affected by attenuation into the respective shots not affected by attenuation
4. Filter the recorded shots with the corresponding matching filters.
5. Migrate the filtered recorded data using RTM without \( Q \).

In the following sections we demonstrate the effectiveness of our algorithm with synthetic and real data examples.

**Synthetic example – the Marmousi model**

We generated a synthetic dataset with a smoothed version of the Marmousi velocity model (Versteeg, 1994), as depicted in figure 1a. The reflected events are generated by the publicly, and jointly available, Marmousi density model. This procedure mimics the general imaging procedure. That is, imaging events generated by contrasts in lithology utilizing a smooth tomographic velocity model. We generated a dataset without considering intrinsic attenuation, and another considering intrinsic attenuation. Henceforth, and in this section only, we will refer to the former as dataset 1, and to the latter as dataset 2. The model of the quality factor utilised in this example is shown in figure 1b. Figure 2a, shows the
RTM image resulting from migrating dataset 1. This figure is the benchmark as it renders an image that is obtained in the perfect case when anelastic loss is not present. Figure 2b, shows the RTM image obtained by migrating dataset 2 without considering any compensation for Q. As can be seen the image obtained without compensation shows the effect of attenuation manifested by the strong decay of amplitude, and evidence of unfocused events with depth. Figure 2c, shows the RTM image obtained by migrating dataset 2 utilizing the compensation algorithm described in the previous section. The image shown in figure 2c is almost identical to that depicted in figure 1a, regarding both the focusing of events and balance of amplitude with depth, thus demonstrating the effectiveness of the compensation algorithm outlined.

![RTM image](image)

**Figure 1** – a) Smoothed Marmousi velocity model; b) Quality factor Q.

![RTM images](image)

**Figure 2** – a) Migration of non-attenuated synthetic data; b) Migration of attenuated synthetic data without Q compensation; c) Migration of attenuated synthetic data with Q compensation.

**Example – Field data**

Finally, we show a straightforward application of the attenuation compensation algorithm to real data. The field dataset is a 2D line recorded off the West coast of Australia. For a comprehensive description of the acquisition parameters see Agudo et al. (2018). This dataset is significantly affected by seismic anisotropy. In all the examples presented in this section, seismic forward- and reverse-modelling are carried out considering seismic anisotropy. We migrated the seismic data with RTM after a conventional pre-processing flow. Figure 3a, shows the resulting migration image without applying compensation for intrinsic attenuation. As can be seen, the shallower sub-horizontal geological units are relatively well resolved. However, the image of the deeper rifted blocks and faults is poor. In addition, it is clear that amplitude weakens with depth. Then, we migrated the dataset utilizing the Q-compensated RTM procedure outlined. Figure 3b, depicts the resulting Q-compensated seismic image showing a significant improvement especially on the resolution of the deeper rifted blocks, definition of faults, and shallower sub-horizontal units, as highlighted by the white arrows. In addition, it improved the balance of the seismic amplitude with depth.
Conclusions

We have introduced an approach for Q-compensated RTM which filters out the effect of anelastic loss from the data. It requires computing a filter matching attenuated to non-attenuated synthetic data. As the filter is computed from synthetic data it contains all the accumulated effects of the propagation path and heterogeneity of the model. In addition, the approach is robust and independent of any complexities in the medium (e.g. anisotropy). Effectively, determining the matching filters depends only on the availability of adequate algorithms for forward modelling and filter estimation. The synthetic and real data examples demonstrate the effectiveness of our method for imaging complex structures with data affected by anelastic loss. Finally, although not demonstrated here, the Q-compensation method outlined can be extended to Q-compensated viscoelastic RTM of multicomponent data.

Figure 3 – Migration of the real data set a) without Q compensation, and b) with Q compensation.

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References

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