Elastic wave velocities in sandstone often show stress-induced anisotropy under nonhydrostatic stress conditions. It has also been recognized that the elastic anisotropy has impact on the AVO (Amplitude Variation with Offset) response (e.g., Blangy, 1994) and imaging velocity (e.g., Thomsen, 1986). It is therefore necessary to assess the impact of in-situ stress conditions on the elastic anisotropy. Moreover, stress-induced anisotropy is a function of reservoir pore-pressure depletion because of changes in triaxial stress state. For example, pore-pressure depletion in a flat reservoir can be reasonably approximated as uniaxial compaction, in which effective horizontal stress change is smaller than the effective vertical stress change.

In this paper, applicability of two anisotropic rock physics models to the prediction of time-lapse changes in elastic anisotropy of tight sandstone was investigated. It was found that these models require only two parameters to reasonably predict the time-lapse change. These two parameters can be estimated without using stress sensitivity of off-axis velocity and even without using that of S-wave velocities. This simplifies the anisotropic modelling process and possibly reduce the number of required measurements. Anisotropic post-production log generation and seismic forward modelling were also performed to demonstrate the practical use of the anisotropic rock physics model.

The anisotropic rock physics models

The model proposed by Sayers and Kachanov (1995) assumes that the stress dependence of the elastic properties of a sandstone is due to the presence of discontinuities within the rock such as grain boundaries and microcracks. It is assumed that the discontinuities are locally flat, and there is rotational symmetry around the normal to the discontinuities. Traction is continuous, but displacement may be discontinuous across the discontinuities. Using Gauss’s theorem, the macroscopic strains can be expressed in terms of displacement jumps at discontinuities. Introducing a symmetric second-order compliance tensor that expresses the vector of average displacement jump in terms of the average stress field (Kachanov, 1992), the macroscopic strain $\bar{\varepsilon}_{ij}$ may be written as:

$$\bar{\varepsilon}_{ij} \equiv \left( S_{ijkl}^0 + \Delta S_{ijkl} \right) \bar{\sigma}_{ij}$$

where $\bar{\sigma}_{ij}$ represents the macroscopic stress components, $S_{ijkl}^0$ is an elastic compliance tensor of the rock without any discontinuities, and $\Delta S_{ijkl}$ is the excess compliance due to the discontinuities:

$$\Delta S_{ijkl} = \frac{1}{4} \left( \alpha_{ijkl} \alpha_{jl} + \alpha_{ijl} \alpha_{jk} + \alpha_{jik} \alpha_{il} + \alpha_{ijkl} \right) + \beta_{ijkl}$$

$\alpha_{ij}$ is a second-rank tensor and $\beta_{ijkl}$ is a fourth-rank tensor. These tensors quantify the impact of the normal and shear compliances of the discontinuities and their orientational distribution on the elastic anisotropy. $\alpha_{ij}$ is a function of the shear compliance, and $\beta_{ijkl}$ is a function of the shear and normal compliances. If these compliances are equal for all discontinuities, $\beta_{ijkl}$ vanishes. Sayers (2002) shows an example in which elastic anisotropy of Penrith sandstone is fitted to reasonable accuracy by the model. In this study, the elastic compliance tensor at the reference stress condition (the highest applied stress) is used as $S_{ijkl}^0$, and transverse isotropy with axis of rotational symmetry along $x_3$ was assumed (i.e., $\alpha_{11} = \alpha_{22}$ and $\beta_{1333} = \beta_{2233}$).

The second model is proposed by Fjaer (2006). The model assumes that the rock can be reasonably approximated as a solid material, with pores and three sets of flat cracks oriented normal to the principal stresses. Each crack set is characterized by a crack density, and the cracks are assumed to be sufficiently flat so that the porosity associated with the cracks can be ignored. Assuming that the cracks are non-interacting, the elastic stiffness tensor elements are given as:

$$C_{11} = C_{11}^0 \left[ 1 - Q_{33} \zeta_1 - Q_{11} (\zeta_2 + \zeta_3) \right]$$

where

- $C_{11}$ is the elasticity tensor
- $C_{11}^0$ is the elasticity tensor of the intact medium
- $Q_{33}$, $Q_{11}$ are the normalized crack densities
- $\zeta_1$, $\zeta_2$, $\zeta_3$ are the normalized crack aspect ratios
\[
C_{33} = C_{33}^0 [1 - Q_{33} \zeta_3 - Q_{11} (\zeta_1 + \zeta_3)]
\]
(4)
\[
C_{12} = C_{12}^0 [1 - Q_{13} (\zeta_1 + \zeta_2) - Q_{12} \zeta_3]
\]
(5)
\[
C_{13} = C_{23}^0 [1 - Q_{13} (\zeta_1 + \zeta_2) - Q_{12} \zeta_3]
\]
(6)
\[
C_{44} = C_{44}^0 [1 - Q_{44} (\zeta_2 + \zeta_3) - Q_{66} \zeta_1]
\]
(7)
\[
C_{66} = C_{66}^0 [1 - Q_{44} (\zeta_2 + \zeta_3) - Q_{66} \zeta_3]
\]
(8)

where \( \zeta_1 \), \( \zeta_2 \), and \( \zeta_3 \) are crack densities. The tensor elements \( C_{ij}^0 \) represent the stiffness of the rock in absence of cracks. The \( Q \)'s are functions of the Poisson’s ratio of the background rock (Hudson, 1981). Fjaer (2006) suggests theoretical expressions for the stress sensitivity of the crack densities and shows that this model reasonably matches stress dependence of anisotropic velocities observed in laboratory tests with dry Castlegate sandstone. In this paper, the elastic stiffness at the reference stress condition (the highest applied stress) was used as \( C_{ij}^0 \) and transverse isotropy with axis of rotational symmetry along \( x_3 \) was assumed (i.e., \( \zeta_1 = \zeta_2 \)). Note that the rock is slightly anisotropic at the reference stress condition. However, since the anisotropy is small, this does not cause significant problems. The Poisson’s ratio was therefore calculated by using the average of vertical and horizontal velocities. If the rock is strongly anisotropic at the reference stress condition, such as shale, the model for anisotropic background should be used (e.g. Guo et al., 2019).

**Model vs measurements**

Applicability of the anisotropic models to the ultrasonic velocity measurements of Knapp et al. (2017) and Asaka et al. (2018) on dry tight sandstone measured in a triaxial frame was investigated. Quantitative evaluation of minerals by scanning electron microscopy results reveals that quartz is the dominant mineral (about 86%) and kaolinite is second in abundance (about 3.4%). The porosity range is 8.3% - 16.7%. Velocities at multiple orientations were measured simultaneously by fitting the core plugs with a rubber sleeve that contained embedded piezometers. All measured velocities were assumed to be anisotropic phase velocities rather than anisotropic group velocities because of small anisotropy of the tight sandstone. Hysteresis tests and a comparison of measured core velocity with sonic log suggested that the effect of grain contacts or microcracks that have opened up during core recovery and that are not open in situ is not significant (Asaka et al., 2018). For vertical plugs, velocities were measured under two different stress conditions: hydrostatic and nonhydrostatic. The nonhydrostatic stress conditions simulate in-situ stress conditions and uniaxial compaction (Asaka et al., 2018). In this paper, 19 vertical plugs were used to investigate the applicability of the anisotropic rock physics models.

Figure 1 (top) shows the results of a least-squares inversion of \( a_{11} \) and \( a_{33} \), assuming that \( \beta_{ijk} = 0 \), using stress sensitivities of only vertical and horizontal P-wave velocities measured under the nonhydrostatic stress conditions \( (a_{11} \) and \( a_{33} \) were determined so that these minimize the difference between the predicted and measured P-wave velocities). Predictions of stress dependencies, shown by solid lines, for oblique P-wave and S-wave velocities are excellent despite the fact that stress dependencies of those velocities are not used for the inversion. Correspondingly, stress sensitivity of anisotropy parameter \( \delta \), which is a difficult parameter to measure experimentally, is predicted to reasonable accuracy. Figure 1 (bottom) shows the results of a least-squares inversion of \( \zeta_1 \) and \( \zeta_3 \), again using stress sensitivities of only vertical and horizontal P-wave velocities. The quality of predictions are similar to that of the Sayers and Kachanov model.

As demonstrated in Figure 1 (top), the contribution of the fourth-rank tensor \( \beta_{ijk} \) can be ignored in this case. This is probably because of small Poisson’s ratio of the dry tight sandstone (~ 0.14 at the reference stress condition). As mentioned in Sayers (2002), the normal and shear compliances of the discontinuities are almost the same (hence the fourth-rank tensor becomes vanishingly small) if Poisson’s ratio of the background medium is small. The fact that stress dependencies of \( a_{11} \) and \( a_{33} \) are similar to that of crack densities \( (\zeta_1 \) and \( \zeta_3 \) ) is because \( a_{11} \) is strongly influenced by shear.
compliance and the number of vertical discontinuities, while $\alpha_{33}$ is strongly influenced by those of horizontal discontinuities.

**Figure 1** Results of the least-squares inversion of $\alpha_{11}$ and $\alpha_{33}$ (top) and $\xi_1$ and $\xi_3$ (bottom) using stress sensitivities of only vertical and horizontal P-wave velocities under the nonhydrostatic stress conditions. Circles in velocities and Thomsen’s anisotropy parameters (Thomsen, 1986) plots are measurements, while solid lines are predictions based on the inverted $\alpha_{11}$ and $\alpha_{33}$ or $\xi_1$ and $\xi_3$.

Anisotropic post-production log generation and seismic forward modelling

Anisotropic post-production log generation and seismic forward modelling were performed using the Sayers and Kachanov model. As demonstrated in Figure 1, only two parameters, $\alpha_{11}$ and $\alpha_{33}$, are required to describe the time-lapse changes in anisotropy in this case. The post-production log generation was performed by defining those two parameters as a logarithmic function of the vertical effective stress, using reservoir simulation data as input data, and assuming uniaxial compaction. Pore-pressure change is large, but the saturation change is negligible in the reservoir simulation data used for the modelling (simulation of gas production). The anisotropic seismic forward modelling was conducted by using the exact VTI coefficients equations (Daley and Hron, 1977). The result is shown in Figure 2. Notice the time-lapse changes in reservoir elastic anisotropy, which makes AVO response different from that based on the isotropic assumption.

Conclusions

It was demonstrated that the two anisotropic rock physics models can be practically used for the modelling of time-lapse changes in elastic anisotropy, which is required to correctly model the 4D seismic response. These models can simplify the modelling process and possibly reduce the number of required measurements.

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**Figure 2** Results of the anisotropic post-production log generation and seismic forward modelling using the Sayers and Kachanov model. P-impedance (Ip), Vp/Vs, delta and epsilon logs are after the 15 m anisotropic Backus averaging.

**References**


