Introduction

The existence of topography in seismic survey areas makes it necessary for researchers to consider the effects on seismic wavefield propagation in seismic wavefield simulation. The amplification effect of the topography has long been investigated by numerous researchers. Numerical methods to tackle with the topographic effect include the finite-element method (Zhou and Greenhalgh, 2011; Zhou et al., 2012; Zhao et al., 2017), finite-volume method (Brossier et al., 2008), and finite-difference method (Rao and Wang, 2013, 2018). The limitation of the finite-difference method, which can only be applied to rectangle grid, renders its application to seismic wavefield simulation in the presence of topography. Rao and Wang (2013, 2018) and Zhao et al (2020) applied the boundary conforming method to deal with the effect of the topography in seismic wavefield simulation. However, these researches only considered the elastic model, while ignoring the attenuating effect of the real earth.

In this research, we combine the attenuating effect and the existence of the topography to frequency-domain seismic wavefield simulation. Complex frequency-shifted perfectly matched layer (CFS-PML) is applied to the seismic simulation to absorb the artificial reflection caused by the limited computational size. Numerical example is used to verify the validity of the proposed algorithm.

Frequency-domain wave equation and free-surface boundary condition

The frequency-domain elastic wave equation can be expressed as

\[-\rho \omega^2 u = \frac{\partial}{\partial x} \left( \lambda + 2\mu \right) \frac{\partial u}{\partial x} + \lambda \frac{\partial^2 u}{\partial x^2} + \frac{\partial}{\partial z} \left( \mu \frac{\partial u}{\partial x} + \mu \frac{\partial u}{\partial x} \right), \quad (1a)\]
\[-\rho \omega^2 v = \frac{\partial}{\partial z} \left( \lambda + 2\mu \right) \frac{\partial v}{\partial z} + \lambda \frac{\partial^2 v}{\partial z^2} + \frac{\partial}{\partial x} \left( \mu \frac{\partial v}{\partial z} + \mu \frac{\partial v}{\partial x} \right), \quad (1b)\]

where \(u\) and \(v\) are the temporal Fourier components of horizontal and vertical displacements, \(\rho\) is the bulk density of the media, \(\lambda\) and \(\mu\) are Lamé constants. Frequency-domain elastic wave equation can be easily extended to frequency-domain visco-elastic wave equation by converting real-valued velocity to complex-valued velocity using Kolsky’s attenuation-dispersion model expressed by

\[\tilde{c}(\omega) = c \left[ 1 + \frac{1}{\pi Q} \log\left( \omega/\omega_r \right) \right] + i \frac{\text{sgn}(\omega)}{2Q}, \quad (2)\]

where \(\tilde{c}(\omega)\) is the complex-valued P- or S-wave velocity, \(Q\) is the quality factor, \(\omega_r\) is the reference frequency, \(\text{sgn}\) is the sign of a number, and \(i = \sqrt{-1}\) (Kolsky, 1953; Wang, 2008, p.19).

![Figure 1 Mapping between computational space and physical space](Image)

The mapping relation between computational and physical domain, which is shown in figure 1, is for transferring the elastic wave equation (1) from the Cartesian domain to curvilinear domain. Then, the frequency-domain seismic wave equation can be expressed in the curvilinear coordinate as
\[-\omega^2 \rho u = q_x \cdot \frac{\partial}{\partial q} \left[ (\lambda + 2\mu)(q_x \partial_q + r_z \partial_r)u + \lambda (q_z \partial_q + r_x \partial_r)v \right] + q_z \cdot \frac{\partial}{\partial q} \left[ \mu (q_x \partial_q + r_z \partial_r)v + \mu (q_z \partial_q + r_x \partial_r)u \right] + r_z \cdot \frac{\partial}{\partial r} \left[ (\lambda + 2\mu)(q_z \partial_q + r_x \partial_r)u + \lambda (q_x \partial_q + r_z \partial_r)v \right] + r_x \cdot \frac{\partial}{\partial r} \left[ \mu (q_x \partial_q + r_z \partial_r)v + \mu (q_z \partial_q + r_x \partial_r)u \right] \]

\[-\omega^2 \rho v = q_x \cdot \frac{\partial}{\partial q} \left[ \mu (q_x \partial_q + r_z \partial_r)v + \mu (q_z \partial_q + r_x \partial_r)u \right] + q_z \cdot \frac{\partial}{\partial q} \left[ (\lambda + 2\mu)(q_z \partial_q + r_x \partial_r)u + \lambda (q_x \partial_q + r_z \partial_r)v \right] + r_z \cdot \frac{\partial}{\partial r} \left[ \mu (q_x \partial_q + r_z \partial_r)v + \mu (q_z \partial_q + r_x \partial_r)u \right] + r_x \cdot \frac{\partial}{\partial r} \left[ (\lambda + 2\mu)(q_z \partial_q + r_x \partial_r)u + \lambda (q_x \partial_q + r_z \partial_r)v \right],

\text{(3a)}

where \( q \) and \( r \) are spatial coordinates in the curvilinear coordinate system, \( q_x, q_z, r_x \) and \( r_z \) denote \( \partial q(x,z)/\partial x, \partial q(x,z)/\partial z, \partial r(x,z)/\partial x \), and \( \partial r(x,z)/\partial z \), respectively.

The corresponding free-surface boundary condition in the curvilinear coordinate is expressed as

\[
(\lambda + \mu) r_x r_z \frac{\partial v}{\partial r} + \left[ \mu q_x r_z + \lambda q_z r_x \right] \frac{\partial v}{\partial q} = 0
\]

\[
(\lambda + \mu) r_z^2 + (\lambda + 2\mu) r_x^2 \frac{\partial v}{\partial r} + \left[ \mu q_z r_x + \lambda q_x r_z \right] \frac{\partial v}{\partial q} = 0
\]

\[
(\lambda + \mu) r_z^2 + (\lambda + 2\mu) r_x^2 \frac{\partial u}{\partial r} + \left[ \mu q_z r_x + \lambda q_x r_z \right] \frac{\partial u}{\partial q} = 0
\]

\text{(4b)}

In the frequency domain, the Ricker wavelet is used as a source signature (Wang 2015):

\[
S(\omega) = \frac{2}{\sqrt{\pi}} \frac{\omega^2}{\omega_p^2} \exp \left[ -\left( \frac{\omega}{\omega_p} \right)^2 \right],
\]

\text{(5)}

where \( \omega_p \) is the angular peak frequency and \( \omega \) is the angular frequency.

**Wavefield simulation**

We tested the validity of our algorithm with a complex topography. The homogeneous elastic isotropic model has a surface of three hills, which is shown in figure 2. The model has a horizontal size of 2.1 km. The surface is represented by the function:

\[
y = 0.1 \times \exp \left[ \left( \frac{x - 0.6}{0.20} \right)^2 \right] + 0.2 \times \exp \left[ \left( \frac{x - 1.8}{0.20} \right)^2 \right] + 0.1 \times \exp \left[ \left( \frac{x - 3.0}{0.20} \right)^2 \right]
\]

The model is discretized by 1400 \times 700 irregular grids. The P-wave velocity, S-wave velocity and density are: 3000 m/s, 2000 m/s, and 2000 kg/m$^3$. The quality factor for P- and S-wave velocity are:
$Q_p = 40$ and $Q_p = 30$, respectively. The thickness of the CFS-PML absorbing boundary condition is 0.3 km (100 grids). Ricker wavelet with the dominant frequency of 7 Hz is located at (1.11 km, -0.06 km) as a vertical point source (star).

**Figure 2** Sketch of the homogeneous elastic isotropic model with a surface of three hills.

The comparison between real parts of frequency solutions of horizontal displacement of seismic wavefield at 15.1 Hz (top panel) and 30.1 Hz (bottom panel) calculated with elastic (left panel) and visco-elastic (right panel) wave equation are shown in figure 3. The high amplification effect of the topography on surface waves can be easily observed in figure 3. What can be observed in the figure 3 is the attenuating effect in the frequency-domain seismic wavefield simulation in the existence of topography.

**Figure 3** Comparison of real part of frequency solution with 10.1 Hz (top panel) and 30.1 Hz (bottom panel) calculated with frequency-domain elastic (left panel) and visco-elastic (right panel) wave equation.

We take inverse Fourier transform of the frequency solutions to obtain time-domain shot gathers, to better observe the attenuation effect. Figure 4 shows the comparison of horizontal shot gather calculated with elastic (4a) and visco-elastic (4b) wave equation. From the comparison, one can see that not only P- and S- wave get attenuated, but also the surface waves.
Conclusion

In this research, we carried out frequency-domain finite-difference visco-elastic seismic wavefield simulation in the presence of topography using boundary conforming method. To absorb the unwanted artificial reflection, we applied CFS-PML absorbing boundary condition to the computational model. Numerical example in this research verified the validity of the proposed algorithm.

Acknowledgements

The author is grateful to the sponsors of the Centre for Reservoir Geophysics, Imperial College London, for the financial support of a postdocship.

References