Introduction

The seismic signal is inevitably corrupted by noise in the process of transmission, which decreases the quality of seismic data and brings difficulties to subsequent interpretation. Therefore, improving the signal-to-noise ratio (SNR) of seismic data is one of the main tasks of seismic data processing. Scholars have developed various denoising methods such as traditional random noise reduction (Canales, 1984; Harris et al., 1997), curvelet transform denoising (Wang et al., 2016), dictionary learning and denoising (Zu et al., 2019) and low-rank recovery (Chen et al., 2016) etc., with their own advantages and disadvantages, among which total variation-based method has been successfully applied to seismic denoising. It can preserve characteristic image features like edges and suppress noise. However, its staircasing effects will lead to blocky phenomenon in denoised sections, limiting the performance. In order to overcome the staircasing effect of TV regularization, high-order derivatives approaches are invented and applied to the standard TV (Bredies et al., 2010).

Events in seismic data have obvious directionality, while the regular TV regularization only tends to reduce the horizontal and vertical gradients and ignores the structural information contained in the seismic data. Imposing the directional information of seismic events is highly desirable. Bayram and Kamasak (2012) propose a directional TV method (DTV) and applied it to digital image denoising. They only take one single dominant direction into account for the whole image and the performance is poor when addressing the denoising problem for complex-textured seismic data. On this basis, Qu et al. (2019) develop a new DTV constraint based on a rough estimate of the subsurface image. They apply the new DTV algorithm to full waveform inversion and joint migration inversion.

Therefore, in the extend abstract, we introduce the high-order derivatives into DTV methods for seismic denoising. It tends to decrease the gradients in the direction of seismic events and its corresponding orthogonal direction, i.e., the structural and directional information contained in the seismic events are considered. In addition, the proposed method involves higher-order derivatives to combat with staircasing effect. A set of parameters are added to balance derivatives with different orders. The results are satisfactory and the staircasing effect is curbed after introducing the high-order derivatives in the DTV method.

Method

The objective function of TV for seismic denoising contains two terms and it can be expressed as:

$$\min \lambda \|s - \bar{s}\|_1 + \alpha (\|\nabla_x s\|_1 + \|\nabla_y s\|_1),$$

(2)

where, the first term is the fidelity term and the last is regularization term. $s$ and $\bar{s}$ are the denoised and noisy seismic section; $\|\cdot\|_1$ and $\|\cdot\|_2$ denote the L1 and L2 norm; $\lambda$ and $\alpha$ are the weight parameter to balance the corresponding terms; $\nabla_x$ and $\nabla_y$ represent horizontal- and vertical-gradient operator, respectively. However, the conventional TV regularization only tends to reduce the horizontal and vertical gradients and ignores the structural direction. Therefore, it can be improved when addressing the data which have obvious directionality. The objective function of DTV is:

$$\min \lambda \|s - \bar{s}\|_2^2 + \alpha (\|\nabla_1 s\|_2^2 + \|\nabla_2 s\|_2^2),$$

(3)

where, $\nabla_1$ and $\nabla_2$ denote the gradient operators of the dominant direction and the direction perpendicular to the dominant direction, respectively. They are the scaled and rotated version of horizontal and vertical gradient operator and can be expressed as:

$$\left(\nabla_1 s, \nabla_2 s\right)^T = M \left(\nabla_x s, \nabla_y s\right)^T,$$

(4)

where, $M = \Lambda R$. $\Lambda = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}$ and $R = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$ denote the scaling and rotation matrix, respectively. For convenience, let $\nabla s = \left(\nabla_x s, \nabla_y s\right)^T$ and $\|\nabla s\|_2^2 = \|\nabla_x s\|_2^2 + \|\nabla_y s\|_2^2$, then the
regularization term of DTV can be represented as: \( \| \nabla s \| + \| \nabla^2 s \| = \| M \nabla^3 s \| \). As the new method involves high-order derivatives, the objective function becomes:

\[
\min \lambda \| s - \bar{\mathbf{s}} \|_2^2 + \sum_{q=1}^{Q} \alpha_q \| \mathbf{M} \nabla^q \mathbf{s} \|_1
\]

(5)

where, subscript \( q \) denotes the \( q \)th order derivatives and \( Q \) is the highest order. \( \alpha_q \) represents the corresponding weights of different orders. If we set \( Q=1, m_1 = m_2 = 1 \) and \( \theta=0 \), the function 4 degenerate into function 1. That is, the conventional TV is a special case of the high-order DTV. For estimating the directional information, we follow the strategy proposed by Lellmann and Morel (2013). Here we do not describe the theory of finding direction information. Similar with the DTV, the objective function 4 can also be easily solved by the primal-dual algorithm (Chambolle and Pock, 2011), thus we use the primal-dual algorithm to minimize the objective function and obtain the denoised section.

**Model data test**

The first example is a model dataset containing three seismic events. This noisy prestack data are synthetized with a 30 Hz Ricker wavelet and contain 300 traces with sampling interval of 4 ms. The noise obeys a mixed distribution with combination of Gaussian, Rayleigh and uniform distribution. The clean and noisy seismic data are shown in sub-figures 1a and 1b. The SNR of the input noised seismic data is -3 dB. Sub-figures 1c-1e display the denoised results by using conditional TV, DTV and high-order DTV approaches. For DTV and high-order DTV, the matrix of \( \mathbf{M} \) is identical. Because the computational time will increase with increasing of the highest order, we set \( Q=3 \) in this example. \( m_1 = 0.21, m_2 = 0.08 \). The output SNR of the three method is 7.02 dB, 8.58 dB and 9.34 dB, respectively. The classic TV and DTV cannot remove the noise very well, where the staircasing effect are also visible. Obviously, the high-order DTV performs best in the three methods. It can be noted that a cleaner section is output by the proposed method and the staircasing effect is weaken. The local similarity map is a valuable tool for detecting signal leakage and denoising behaviour (Chen and Fomel, 2015). After using the new method, the coherent signal disappears in the final noise section and the useful signal leakage closes to zero in the local similarity map, indicating that it can remove noise and protect effective signal. In addition, we access the performance of these methods on this model data by noise with different SNR. The output SNR are listed in the Table 1.

<table>
<thead>
<tr>
<th>Input SNR</th>
<th>-6 dB</th>
<th>-4 dB</th>
<th>-2 dB</th>
<th>0 dB</th>
<th>2 dB</th>
<th>4 dB</th>
<th>6 dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output SNR by different method</td>
<td>TV</td>
<td>5.25 dB</td>
<td>6.18 dB</td>
<td>7.98 dB</td>
<td>8.55 dB</td>
<td>10.03 dB</td>
<td>12.47 dB</td>
</tr>
<tr>
<td></td>
<td>DTV</td>
<td>6.40 dB</td>
<td>7.47 dB</td>
<td>9.11 dB</td>
<td>10.34 dB</td>
<td>11.64 dB</td>
<td>13.96 dB</td>
</tr>
<tr>
<td></td>
<td>New DTV</td>
<td>7.52 dB</td>
<td>8.23 dB</td>
<td>10.01 dB</td>
<td>10.97 dB</td>
<td>12.10 dB</td>
<td>14.31 dB</td>
</tr>
</tbody>
</table>

**Field data application**

Then, we apply the three regularization approaches to a field dataset that is a stacked seismic section (as shown in figure 2a) from marine data. It has 400 traces in the horizontal direction. The time ranges from 2 s to 4 s with a sampling interval of 4 ms. It is worth mentioning that the valid signals are completely submerged by the noise. We use classic TV, DTV and the high-order DTV to attenuate the random noise, respectively. The corresponding denoised results are displayed in sub-figures 2b-2d. The performance of conventional TV and DTV are inferior to the high-order DTV in suppressing noise. The noise has been suppressed and events become visible after applying the proposed method. In addition, classic TV and DTV cause more serious damage to the effective seismic signal than the proposed method. As observed from sub-figures 2h-2j, the local similarity section of high-order DTV is closer to zero than that of the other two methods. Figure 3 gives the zoomed sections corresponding to the red box in figure 2a, it can be seen that the new method gives a more promising result either in attenuating noise or in protecting the useful seismic signal.
Conclusions

We describe a high-order directional total variation method for seismic denoising. The method considers the directional information contained in the seismic data and tends to reduce the gradients on the dominant direction and its corresponding orthogonal direction. The performance is improved by introduce the high-order derivatives to overcome the staircasing effect that limits the application of TV-based denoising method. The calculation time will be prolonged when a higher order is set. The proposed method output clean seismic sections with less signal leakage in the both examples, which
indicates the potential of the method. A more effective strategy to project the useful signal will be studied in the future. In addition, the method can be generalized to an anisotropy high-order DTV denoising method by adding an anisotropy operator on the derivative operator when addressing complex seismic denoising problem.

Figure 3 Zoomed section corresponding to the frame box in figure 2. (a) is the noisy seismic data; (b)-(d) display denoised results by using TV, DTV and high-order DTV, respectively; (e)-(g) are the corresponding noise sections; (h)-(j) show the corresponding local similarity sections.

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References