Introduction

Cold seepage is a phenomenon involving emissions of methane-rich fluid from marine sediments, occasionally accompanied by heavier hydrocarbons and CO2, and the fluid temperature is close to the surrounding seawater (Ye et al., 2019). It’s closely related to natural gas hydrates which are commonly distributed along continental margins (Zhang et al., 2019). Bubble plume has potential impacts on the study of gas hydrates development, marine engineering safety, unconventional oil and gas exploration (Fujikura et al., 2017). As a sign of the distribution of natural gas hydrates, cold seepage bubble plume has been attracting extensive attention to international academia. So far, there are over 900 cold seepage activity areas including the Arctic shelf NWSvalbard around the world. At present, the side-scan sonar, single-beam sonar, and multibeam sounding are three main ways to detect cold seepage, which can clearly identify the bubble plume. Unlike those small-scale detection methods, the seismic method is a more large-scale and economical choice to regard plume bubbles as scattering points since seismic acoustic source wavelength is much larger than the scale of a plume bubble. Li et al. (2013) built an acoustic bubble plume model based on random medium theory and obtained seismic responses by finite-difference forward modelling.

In this paper, the Keller-Miksis bubble vibration model is used to describe the movement state of cold seepage bubble under the action of seismic wave, and the acoustic interaction between bubbles is considered to build the model. We use the bubbly liquid acoustic equation to describe the propagation of acoustic wave in the seafloor cold seepage. Then the attenuation coefficient and velocity are derived to realize the accurate numerical simulation of the seafloor cold seepage plume flow. We use a 9-point frequency-space-domain FD scheme for acoustic wave equation forward modelling. Numerical tests show that the established model can accurately describe cold seepage bubble plume, the scattering wave series of seismic responses have obvious directionality along the extension direction of bubble plume and the scattering region is coinciding with the shape of the bubble plume water body.

Method

When acoustic wave propagates in the liquid containing bubbles, using \( \rho, u, P \) to represent the density, velocity field of particle vibration and wavefield of the liquid respectively, the continuity equation in the liquid containing bubbles is

\[
\frac{1}{\rho v^2} \frac{\partial P}{\partial t} + \nabla \cdot u = \frac{\partial \beta}{\partial t},
\]

where \( v \) represents velocity, \( \beta \) represents the total volume of bubbles in the mixture as a percentage of the mixture volume and its expression is as follows:

\[
\beta = \frac{3}{4} \pi R^3 N,
\]

where \( R \) represents the transient radius of bubbles, and \( N \) represents the number of bubbles in a unit volume of liquid and it is a constant. Considering the momentum equation:

\[
\rho \frac{\partial u}{\partial t} + \nabla P = 0.
\]

Through equations (1) and (3), the propagation equation of acoustic wave in bubble liquid can be obtained:

\[
\frac{1}{v^2} \frac{\partial^2 P}{\partial t^2} - \nabla^2 P = \rho \frac{\partial^2 \beta}{\partial t^2}.
\]

We use equation (4) to describe the propagation of acoustic waves in bubbly liquid. In the case of linear response, the radial variation of bubble radius can be expressed as

\[
R = R_0 + \epsilon (t),
\]

where \( R_0 \) is radius of the bubble without acoustic pressure driving; \( \epsilon (t) \) is the change of the radius of the bubble under the action of acoustic wave, which is a function of time.
According to the Keller-Miksis equation, the radial vibration of bubble considering the compressibility of liquid can be described as:

\[
\left(1 - \frac{1}{v} \frac{\partial R}{\partial t}\right) \rho \left(\frac{\partial^2 R}{\partial t^2}\right) + \frac{3}{2}\left(1 - \frac{1}{v} \frac{\partial R}{\partial t}\right) \left(\frac{\partial R}{\partial t}\right)^2 = \frac{1}{\rho} \left(1 + \frac{\partial R}{v} \frac{\partial}{\partial t} + R \frac{\partial}{\partial t}\right) (P_s - P) \tag{6}
\]

where \( P_s = P_s(R, t) - \frac{4\eta}{R} \frac{\partial^2 R}{\partial t^2} \frac{2\sigma}{R} \) is the wavefield in the liquid outside the bubble, \( P_g(R, t) \) is the wavefield inside the bubble, and \( \sigma \) is the surface tension coefficient of the liquid.

Considering equation (5), equation (6) becomes:

\[
\frac{1}{v^2} \frac{\partial^2 P}{\partial t^2} - \nabla^2 P = 4\pi \rho N \left(2R \left(\frac{\partial R}{\partial t}\right)^2 + R^2 \frac{\partial^2 R}{\partial t^2}\right) \tag{7}
\]

Considering the absorption effect when acoustic wave propagates in the liquid, equation (7) becomes:

\[
\frac{1}{v^2} \frac{\partial^2 P}{\partial t^2} - \nabla^2 P - \delta \frac{\partial^2 P}{\partial t^2} = 4\pi \rho N \left(2R \left(\frac{\partial R}{\partial t}\right)^2 + R^2 \frac{\partial^2 R}{\partial t^2}\right) \tag{8}
\]

In equation (8), \( \delta = \frac{2\nu \alpha}{\omega^2} \), \( \alpha \) represents the absorption coefficient of the liquid to acoustic wave, which meet the equation (9) in water:

\[
\alpha \approx 6.3 \times 10^{-16} \text{s}^2 \text{m}^{-1}, \delta \approx 2.8 \times 10^{-9} \text{m} \tag{9}
\]

Assume the incident plane wave is:

\[
P = P_0 e^{-i(\omega - k \cdot \rho)} \tag{10}
\]

where \( P_0 \) represents the initial wavefield, \( P_s \) represents the seismic wavefield, and \( \omega \) represents the circular frequency of the acoustic wave.

Equations (5) and (10) are substituted into equation (6) to obtain the expression of linear vibration of bubbles:

\[
\frac{\partial^2 \xi}{\partial t^2} + a \frac{\partial \xi}{\partial t} + \omega_b^2 \xi = -P_0 / \left(R \rho + 4\eta / \nu\right) e^{-i(\omega - k \cdot \rho)} \tag{11}
\]

where

\[
a = \left(4(\eta \nu + \sigma) / R_0 + 3P_0\right) / \left(R_0 \rho + 4\eta / \nu\right) \tag{12}
\]

\[
\omega_b = \left[(4\sigma / R_0 + 3P_0) / \left(R_0 \rho + 4\eta / \nu\right) / R_0\right]^{1/2} \tag{13}
\]

\( \omega_b \) is the resonance frequency of the bubble. We assume \( \xi = \xi_0 e^{-i(\omega - k \cdot \rho)} \), and substitute it into equation (10):

\[
\xi_0 = P_0 / \left(R \rho + 4\eta / \nu\right) / \left(\omega^2 - \omega_b^2 + i \omega\right) \tag{14}
\]

From this, the expression of bubble radius \( R \) is known, which is substituted into equation (8), and the expression \( \nabla^2 \mathbf{P} = -k^2 \mathbf{P} \) on seismic wavefield is obtained (Yuan et al., 2018), wherein,

\[
k^2 = \frac{\omega^2}{\nu^2} \left[1 + i\omega \delta / \nu - \left(4\pi NR_0 \nu^2\right) / \left(1 + 4\eta \nu / R_0\right) / \left(\omega^2 - \omega_b^2 + i \omega\right)\right] \tag{15}
\]

The real part \( \text{Re}(k) \) of \( k \) represents acoustic wave propagation, while the imaginary part \( \text{Im}(k) \) of \( k \) represents the absorption coefficient of the bubble liquid to acoustic wave, the propagation velocity of acoustic wave in the cold seepage is given by \( v = \frac{\omega}{k} \).

**Examples**

Besides velocity of the seafloor cold seepage, bubble content, density, viscosity coefficient, surface tension coefficient, and other parameters should be described. We design a complex plume flow model (shown in Figure 1) to test the performance of bubbly liquid acoustic equation. The model includes five
independent cold seepage bubble plumes with different heights and shapes. The model size is 1200 m×600 m, and the velocity background field is 1500 m/s.

**Figure 1** Parameter model of complex seafloor cold seepage. (a)-(e) represent the number of bubbles per unit volume, shear viscosity coefficient, liquid density, surface tension coefficient, and liquid velocity, respectively.

**Figure 2** Snapshots of seafloor cold seepage model. (a) and (b) represent the snapshots simulated by acoustic equation at 0.3 s, 0.4 s, respectively. (c) and (d) represent the snapshots simulated by bubbly liquid acoustic equation at 0.3 s, 0.4 s, respectively.

**Figure 3** Simulated records of seafloor cold seepage model. (a)-(c) represent the records simulated by acoustic equation at 80 Hz, 160 Hz, and 240 Hz dominant frequency, respectively. (d)-(f) represent the records simulated by bubbly liquid acoustic equation at 80 Hz, 160 Hz, and 240 Hz dominant frequency, respectively.
In this paper, a typical seafloor cold seepage bubble plume model was simulated by a 9-point frequency-space-domain finite-difference scheme in different dominant frequencies, and corresponding wavefield snapshots and seismic records were obtained. The grid interval is 1 m and time interval is 0.2 ms. The source is Ricker wavelet with different dominant frequencies of 80, 160 and 240 Hz at the location of (600 m, 20 m), and geophones are set on the sea surface.

It can be seen that seismic wavefield appears obvious disorder and scattering characteristics after encountering cold seepage bubbles. As the wavefield continues to propagate downward, the disordered area increases, and the scattering characteristics gradually increase. Comparing the snapshots simulated by different equations in the same simulation conditions, the scattering characteristic of the latter in the cold seepage area is clearer than the former, which can describe the scattering effect of cold seepage bubble under the action of acoustic wave more particularly. The single shot records in different dominant frequencies corresponding to above model are displayed in Figure 3, which shows that the seismic wavefield has evident scattering characteristics at the location of the plume and the scattering energy in this area is the strongest. With the increase of the wavelet dominant frequency, the resolution of the seismic records is improved, and the scattering response characteristics of the cold seepage area are clearer. It’s easy to conclude from the comparison between the different equations that the bubbly liquid acoustic equation could describe the bubble movement under the action of acoustic wave more practically.

Conclusions

We use bubbly liquid acoustic equation to describe acoustic wave propagation in cold seepage. Compared with traditional acoustic equation, the new equation can demonstrate the scattering characteristics of cold seepage more accurately: The seismic wavefield shows evident scattering which is the most obvious in the bubbly area with the strongest energy. The scattering energy gradually decreases to both sides, and the downward energy decays fast. The seismic waveform is a series of scattering waves with the location of bubbles as the vertex, and the multi-scattering is coherent to form a scattering region consistent with the shape of cold seepage plume. With the dominant frequency of wavelet rising, the scattering disorder of the wavefield increases gradually, while the resolution of the seismic responses is improved. At the same time, the depiction of the shape and location of the cold seepage is more distinct.

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References