HQS-HRINet: An unrolled deep learning method for seismic high-resolution inversion with an inaccurate wavelet

Introduction

In seismic exploration, the research target zones have become more and more complex, so that higher exploration accuracy is required to characterize the geological structures, lithological changes, fluid properties, etc. High-resolution inversion is an effective method to improve the resolution of seismic data. The main idea of the high-resolution inversion is to construct a loss function and then to minimize the loss function for finding an optimal solution. However, because of the band-limited seismic wavelet and the existing noise, the high-resolution inversion problem is usually ill-posed. The most common strategy is to adopt the regularization method, where some constraints are imposed on the estimated model parameters. However, the regularization parameters and constraint functions need to be pre-selected, which are subjective and non-trivial. Further, the determination of the hyper-parameters in the optimization algorithm is also challenging. The above limitations lead to a limited performance for the model-driven methods, and the large computational complexity impedes the application of them.

Recently, deep learning methods have been successfully applied in geophysical processing (Gao et al., 2019), which has great learning ability. Based on the advantages of the deep neural networks, we propose a new unrolled iterative deep neural network called HQS-HRINet to implement the band-limited high-resolution inversion with an inaccurate wavelet. The idea of the proposed method is to unroll the half-quadratic splitting (HQS) algorithm into an iterative deep neural network, and use the residual CNN blocks to learn the proximal mapping in HQS algorithm, which can avoid the determination of regularization parameters and constraint functions. Before inversion, the first thing is to estimate an accurate wavelet. Generally, the most easiest way is to estimate a zero-phase wavelet by amplitude spectral fitting. However, this estimated zero-phase wavelets are usually not accurate, especially in the case of deviating from some assumptions in the spectral fitting methods. In this paper, we apply the error back-propagating method to compensate the errors caused by the inaccurate wavelet, which can make the proposed method applied to the seismic data after zero-phase correction better.

Theory

The band-limited high-resolution inversion frame can be expressed as

\[ J_{\tilde{r}} = \min_{\tilde{r}} \frac{1}{2} \| \tilde{y} - W\tilde{r} \|_2^2 + \mu Q(\tilde{r}), \]  

(1)

where \( \tilde{y} \) indicates the filtered observation by a wide band wavelet \( w_b(t) \), the convolution matrix \( W \) consists of seismic wavelet \( w(t) \), and \( \tilde{r} \) is the filtered reflectivity (band-limited high-resolution result) by a wide band wavelet \( w_b(t) \). Besides, \( \mu \) is the regularization parameter, and \( Q \) is the constraint function. To solve formula 1, the regularization parameter and constraint function need to be pre-determined. To address the above issues, we incorporate the HQS algorithm with deep neural network by mapping each iteration of the iterative HQS algorithm into a layer of the deep network architecture, where the proximal operator is learned by the residual CNN. To introduce our proposed HQS-HRINet, we first introduce an auxiliary variable \( z \), and the optimization problem 1 is reformulated as

\[ J_{\tilde{r}, z} = \min_{\tilde{r}, z} \frac{1}{2} \| \tilde{y} - Wz \|_2^2 + \beta z - \tilde{r}_{k-1} \|_2^2, \]  

(2)

where \( \beta \) denotes a penalty parameter. To obtain the solution of formula 2, we split it into the following sub-problems,

\[ z^k = \arg \min_z \frac{1}{2} \| \tilde{y} - Wz \|_2^2 + \beta z - r^{k-1} \|_2^2, \]  

(3)
\[ \tilde{r}^k = \arg \min_{\tilde{r}} \mu Q(\tilde{r}) + \beta \| z^k - \tilde{r} \|_2^2, \]  

(4)

where the superscript \( k \) refers to the \( k \)th iteration. For formula 3, to avoid the product and inverse of a matrix, the following closed form solution is derived,

\[ z^k = F^H(\frac{F(w)^* \cdot F(\tilde{y}) + \beta^k F(\tilde{r}^{k-1})}{F(w)^* \cdot F(w) + \beta^k}), \]  

(5)

where \( F \) and \( F^H \) indicate the Fourier transform and its inverse transform, respectively, \( w \) is the seismic wavelet, and symbols \( * \) and \( \cdot \) are the conjugate transpose and dot product. Unlike some conventional regularization methods, the parameter \( \beta \) varies with the number of iterations, which is treated as a weight of deep neural network and learned by training data. For formula 4, reduce it into the proximal version,

\[ \tilde{r}^k = \text{prox}_{\beta^k g}(z^k, \tilde{r}^{k-1}), \]  

(6)

It is obvious that the proximal operator \( \text{prox}_{\beta^g} \) represents the mapping between \( z^k \) and \( \tilde{r}^k \) and integrates the previous update \( \tilde{r}^{k-1} \). Here, inspired by the great learning capability of deep learning method, we replace the \( k \)th iteration proximal operator \( \text{prox}_{\beta^g} \) with learning operator \( \Gamma_{\Theta^{k}_{\text{prox}}} \), where the \( k \)th iteration parameters \( \Theta^{k}_{\text{prox}} \) are learned from training sets. The learning operator is implemented by a residual CNN, as shown in Figure 1(a). Finally, the HQS-HRINet network structure is built by unrolling equations 5 and 6, as shown in Figure 1(b), where the stage number corresponding to the iteration number in the HQS algorithm is 3, i.e. \( N_t = 3 \).

The total parameters of this network are expressed as \( \Theta = (\beta^k, \Theta^k_{\text{prox}})_{k=1}^{N_t} \), learned by minimizing the following loss function \( E(\Theta) \)

\[ E(\Theta) = \frac{1}{N} \sum_{i=1}^{N} \| \Lambda_{\Theta}(\tilde{y}_i) - \tilde{r}^{gt}_i \|_2^2, \]  

(7)

where \( N \) is the number of training data pairs, \( \tilde{r}^{gt}_i \) is the ground-truth data for the \( i \)th data pair, and \( \Lambda_{\Theta} \) is the mapping function between the observation data and inverted high-resolution result, learned by the network in Figure 1(b). All the source codes are implemented with the TensorFlow. Note that the seismic wavelet needed in formula 5 is estimated by a simple 21-point smooth function, where the error of the amplitude spectrum will be compensated by the error back propagating. Therefore, the proposed method can be applied to seismic data after zero phase correction.

**Examples**

To train the proposed network, the training sets have to be prepared. Based on the Marmousi II velocity model and density model, the reflectivity can be obtained by the vertical incidence reflectivity formula. Then the desired band-limited high-resolution data (label data) is obtained by convoluting the reflectivity with a wideband wavelet, as shown in Figure 2(c). Besides, a zero-phase Ricker wavelet library is set, where the range of dominant frequency is 30—40 Hz. The training sets are generated by the convolution between the Marmousi band-limited high-resolution data (from trace 400 to 1000 in Figure 2(c)) and the seismic wavelets in wavelet library randomly. Then the proposed network is trained by the training sets.

To test the proposed network, the test data is generated by the convolution between the Marmousi band-limited high-resolution data in Figure 2(c) and a 35Hz Ricker wavelet with zero rotated phase. It means...
that the inversion results including the training result and test result, which can demonstrate the training precision and prediction precision of the proposed network. Figure 2(a) shows the synthetic data where the synthetic data have 1360 traces and 551 samples involved in each trace. Figure 2(b) shows the estimated wavelets just by a simple amplitude spectral fitting. It can be observed that the estimated wavelets are not accurate with some errors. However, the effect of the errors has been considered into the proposed network, so that the errors are corrected and a good inversion result is shown in Figure 2(d). Obviously, the resolution of Figure 2(d) is higher than that of the synthetic data in Figure 2(a), which has clearer structures. To validate the accuracy of the inversion results, we compare it with the true high-resolution data shown in Figure 2(c). It is obvious that these two profiles are similar to each other. It means that the proposed network can reliably expand the frequency band of the original synthetic data in Figure 2(a). Further, the prediction precision and training precision are both high. Therefore, the proposed network has the potential to invert the complex field data.

To further validate the proposed network, we apply it to the field data. Figure 3(a) shows a post-stack field data profile, which is provided by China National Offshore Oil Corporation (CNOOC). The field data has been processed via the conventional processing flows including geometric spreading amplitude compensation, random noise attenuation, stack, and post-stack time migration and so on. It needs to be noticed that the field data has been processed by the phase rotated. Then we input the field data into the trained proposed network, and the obtained inversion result is shown in Figure 3(b). It is obvious that the inversion result has a higher resolution than the original field data. Figure 4 shows the amplitude spectra, where the red dotted line is the average amplitude spectrum of the inversion data, the blue solid line is the average amplitude spectrum of the observed data, and the green solid lines is the spectrum of the wideband wavelet. Through observation, the amplitude spectrum of inversion data is wider than that of the observed data and close to that of the wideband wavelet. Thus, the proposed network can be applied to invert the field data to improve its resolution based on the above analysis.

Conclusions

To realize an automatic high-resolution inversion, an unrolled iterative deep neural network is proposed, where it unrolls the iterative HQS algorithm into a deep neural network and the residual CNN blocks are instead of the original proximal mapping function. The proposed deep learning method is a general inversion frame to implement other similar inversion problems, which can avoid the determination of regularization parameters and constraint functions, and reduce the computation cost. Further, the proposed method can compensate for the error caused by the inaccurate wavelet, so that it can be applied to the field data after zero-phase correction directly. Finally, the synthetic data and field data examples have been conducted to validate the effectiveness of the proposed network. Therefore, the proposed
method has the potential to replace the non-learning methods for realizing the intelligent high-resolution inversion.

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Figure 2 The test of synthetic data, (a) the synthetic 35Hz seismic data, (b) the estimated wavelets by a 21-point smoothing, (c) the true high-resolution data, and (d) the inversion result inverted by the proposed network.

Figure 3 The test of field data, (a) the observed field data and (b) the inversion result by the proposed network.

Figure 4 The amplitude spectra. The red dotted line is the average amplitude spectrum of the inversion data, the blue solid line is the average amplitude spectrum of the observed data, and the green solid lines is the spectrum of the wideband wavelet.

References