Introduction
One of the complexity in simulating the near-well fluid flow may appear in reservoirs with water or gas coning problems. Therefore, the use of fine-scale gridding in space and time is believed to be required for near-wellbore regions simulation (Deimbacher & Heinemann, 1993; Kheriji, Masson, & Moncorgé, 2015). The perpendicular bisector (PEBI) grid is considered to be one of the unstructured grids to modify near-well phenomenon problems (Heinemann, 1988). Multi-scale modeling includes multiple grids at various scales is a common method to solve such problems (Deimbacher & Heinemann, 1993). In this paper, we presented a new method called the Double-Scale method. The proposed Double-Scale method that is based on the overlapping grid method describes radial grids that overlap the Cartesian grid in the near-well region. At the current time, as far as the authors know, the overlapping grid method is not used in the reservoir engineering simulation. The idea of the overlapping grids seems useful for overcoming the problems of the different scales in time and space of the near-wellbore fluid flow. The proposed Double-Scale method is implemented in the Matlab Reservoir Simulation Toolbox (MRST) 2016b, which is an open-source software. This toolbox is created by the finite volume method and the black-oil model (Krogstad et al., 2015). As the authors know, although MRST has various structured and unstructured grids to improve the near-well simulations like PEBI grids does not currently contain a radial grid module. However, generating a large geometry (in field-scale) with several production and injection wells is challenging by using the PEBI grids method. Also, the PEBI grid is not a discrete section in computations that results in increasing the computational costs and the size of the coefficient matrix. In order to show the capability of our Double-Scale method, a three-phase coning case (known as the second SPE comparative study) is selected to simulate using a fine radial grid, a Cartesian grid, the proposed Double-Scale method, and a PEBI grid in MRST.

Method
Figure 1 shows the schematic of pressure distribution versus distance near the wellbore region. As seen, a pressure drops sharply in a small zone around the wellbore with consequences in fluid flow. Likewise, this type of phenomena occurs in case of saturation in coning problems. Generally, using a coarse Cartesian grid to simulat result in some differences between the average saturation/pressure of each block and saturation/pressure at the block center, particularly in the case of well block. Overall, the methods like Peaceman well model (Peaceman, 1978) are suggested to adjust the influence of coarse Cartesian grids. While these methods are believed to have good accuracy in the case of single phase flow problems, e.g., dry gas, the significant errors may occurs in modeling where liquid/gas saturation changes are considerable, e.g., water or gas coning and gas-condensate reservoirs.

![Pressure curve vs. distance from the well](https://via.placeholder.com/150)

**Figure 1** Pressure curve vs. distance from the well

In this proposed method, wells are located at the center of the radial grid which is overlapping only at their blocks of the Cartesian grid. One grid is considered as the base grid (routinely a coarse Cartesian grid), and other grids are overlapped on the base grid where complexity exists. The connection between the overlapped and overlapping grids created by the interpolation of pressure values well blocks and the neighboring blocks on the outer boundary of the radial section.
As mentioned before, the lake of a radial module in the MRST contributes to choose a thin sector of the cylindrical zone around the wellbore (with the Cartesian grid) is modeled to solve the radial grid section in the proposed Double-Scale approach to model near-wellbore phenomena. Five algorithms (D1, D2, D3, N1, and N2) are designed to connect two grids in the proposed Double-Scale method which are categorized based on the type of the outer boundary condition, outer radius length, and the updating saturation and pressure value type. Among them, algorithm D3 provides the best accuracy.

Example
The second SPE comparative solution project (Weinstein, Chappelear, & Nolen, 1986) (SPE2) was selected for comparison between our Double-Scale method and a PEBI grid which is implemented in MRST, in terms of computational cost and accuracy. SPE2 includes a radial reservoir with a single oil production well in the center of the reservoir that was solved as a sector of the reservoir due to the computational costs. Previously, this benchmark problem was solved by eleven companies, and the solutions made available since 1986 by Weinstein et al. (Weinstein et al., 1986).

First, a comparison between the results of four simple cases simulated by both the PEBI grid and the Double-Scale method, and the results were published to those of reference (Weinstein et al., 1986) is obtained to show the accuracy of our model.
1. “Radial 1”: The base SPE2 sector case with 10 blocks in the radial direction (Weinstein et al., 1986).
3. Cartesian grid with 11x11 blocks
4. Cartesian grid with 21x21 blocks

![Figure 1 Water Cut (WCUT) results versus time (the gray range belongs to the results of eleven companies from (Weinstein et al., 1986)).](image)

Overall, as shown in Figure 1 all cases have shown acceptable agreement with the results of (Weinstein et al., 1986). Next, the Cartesian grid (21x21) was chosen as the background grid in both the PEBI and the Double-Scale method. In order to choose the best result, two types of Double-Scale models were examined with 10 and 20 radial block. Also, we examined two types of PEBI, too. Specifications of these cases are given in Table1. The schematic of two types of PEBI which were tested are shown in Figure 2.

Finally, the best ones (DS2 and PEBI1) were chosen and called “Double-Scale” and “PEBI” and the results of water cut and oil production rate are shown in Figure 3 and Figure 4, respectively.
Table 1 Specifications of the studied cases in the three-phase coning problem.

<table>
<thead>
<tr>
<th>Cases</th>
<th>Number of blocks in radial grid</th>
<th>Number of blocks in $\theta$ direction</th>
<th>Inner radius of radial grid (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DS 1</td>
<td>10</td>
<td>1</td>
<td>0.25</td>
</tr>
<tr>
<td>DS 2</td>
<td>20</td>
<td>1</td>
<td>0.25</td>
</tr>
<tr>
<td>PEBI 1</td>
<td>10</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>PEBI 2</td>
<td>10</td>
<td>10</td>
<td>2</td>
</tr>
</tbody>
</table>

Figure 2 Top view of PEBI structure around the well.

Figure 3 Water Cut (WCUT) results compare with reference solution versus time.

Same as previous, the result of the fine-grid is assumed as the reference solution. Generally, both “Double-Scale” and “PEBI” provide the results with good accuracy.

The CPU times of simulations are given in Table 2 which are obtained by an Intel(R) Core (TM) i7-7700K CPU @ 4.20 GHz system. The Fine-grid solution CPU time was 906 seconds and is considered as a reference value. As seen, PEBI grids need more CPU time than the Double-Scale method (about 1.7 times larger).
Figure 4 Oil production results compare with reference solution versus time.

Table 2 CPU times of simulation in seconds for different cases of coning example.

<table>
<thead>
<tr>
<th>Case</th>
<th>Cartesian 21x21 as base grid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Double-Scale</td>
<td>5005</td>
</tr>
<tr>
<td>PEBI</td>
<td>8677</td>
</tr>
</tbody>
</table>

It is also worth mention that here, we used the PEBI grid and the proposed Double-Scale method in a simple example with single production well while in case of the more complex problems, the implementation of our Double-Scale method seems more effortless than using suitable PEBI grids.

Conclusions

The application of the proposed Double-Scale method was shown by simulating the famous three-phase coning benchmark problem (SPE2). This method is applicable in reservoirs zones with intense saturation variations. In comparison to the PEBI grid, our proposed Double-Scale method obtained an acceptable accuracy with almost half CPU time compare with the second SPE comparative study.

References


