Quality control of under-constrained Marchenko equation solvers in complex media using reference focusing functions

Introduction

The Marchenko method is a state-of-the-art approach for seismic data processing and imaging, which removes the influence of the overburden in a data-driven way, yielding target-only reflection data (see Wapenaar et al. (2014) as well as more recent papers referring to it). As the scheme gradually enters the seismic experimental processing deployment stage, algorithm testing involves both simple as well as increasingly more complex synthetics. It is the latter where the Marchenko equation based schemes could benefit from accurate reference solutions for the so-called focusing functions. So far, their quality had been rarely scrutinized in the literature, in particular, because it is currently only known how to find them for horizontally-layered media with a small number of interfaces.

In complex geologies, however, the original formulation by Wapenaar et al. (2014) is no longer properly constrained, and developments of alternative constraints is hindered by our ability to verify the soundness of the obtained solutions. The problem is that accurate quality control to-date was not really possible directly after solving the Marchenko equation. Instead, this is only possible a few the steps later (multidimensional de-convolution, FK-filtering and migration), and each of those can introduce errors and artefacts of their own, rendering further improvements to the method difficult to pin-point and address. In Elison et al. (2020), we have shown that even subtle changes to the solutions to the Marchenko equation can have a significant impact on the target reflection response. Finding a high quality focusing function by means other than the Marchenko equation is therefore pivotal to development and testing of the algorithm in realistically complex scenarios.

We propose to address this problem and model acoustic focusing functions using a frequency-domain depth extrapolation method developed by Kosloff and Baysal (1983). These modelled focusing functions can be used as reference wavefields for the Marchenko-solutions, even for complex geological models that include fine layering and lateral heterogeneities. This gives us confidence to use the results as benchmark for the recently developed extension of the Marchenko method, i.e., the augmented Marchenko method (Dukalski et al., 2019), on a challenging synthetic example studied in its 1.5d extension by Elison et al. (2020). To verify our results we also re-inject these solutions using a time-domain finite difference modeling code to assess the quality of the focusing.

Theory

Numerical modeling of focusing functions presented in this work is based on wavefield depth extrapolation (i.e., propagating a wavefield through subsequent depth levels). Algorithms for such wavefield extrapolation have been available for many years with the main purpose of wavefield extrapolation for seismic migration. Here we rely on the work of Kosloff and Baysal (1983) (KB) since it allows simulation of two-way wave propagation in heterogeneous 2-D and 3-D media in a relatively simple fashion. The acoustic wave equation can be expressed in the space-frequency domain as a set of coupled equations according to

\[
\frac{\partial}{\partial z} \left( \begin{array}{c} P \\ \frac{1}{\rho} \frac{\partial P}{\partial z} \end{array} \right) = \left( \begin{array}{cc} 0 & -\omega^2 \\ -\frac{\partial}{\partial z} \left( \frac{1}{\rho} \frac{\partial}{\partial z} \right) & \rho \end{array} \right) \left( \begin{array}{c} P \\ \frac{1}{\rho} \frac{\partial P}{\partial z} \end{array} \right) \quad \Leftrightarrow \quad \frac{\partial}{\partial z} \bar{P} = \alpha' \bar{P},
\]

where \( P \) denotes acoustic pressure, \( z \) depth, \( \omega \) angular temporal frequency, \( \rho \) mass density and \( c \) seismic velocity, respectively. Note that this method is suitable for laterally varying media, and can be extended to the elastic wave equation. Evanescent waves must be excluded after every extrapolation step to ensure a stable behavior. Using the above equation we can relate the acoustic pressure and to its vertical derivative at different depth levels via \( \bar{P}(z_{j-1}) = (1 + \alpha' / (z_j) \delta z) \bar{P}(z_j) \).

Using definitions of the pressure-normalized down- and upwards travelling components \( P^+ \) and \( P^- \) (Wapenaar, 1998), together with the boundary conditions of the up- and down-propagating components of the focusing functions

\[
\partial_z \left[ \begin{array}{c} F_1^+(x,x_i,\omega) \\ F_1^-(x,x_i,\omega) \end{array} \right] |_{z=0} = \frac{1}{2} i \omega \rho \delta (x - x_j), \quad \partial_z \left[ \begin{array}{c} F_1^+(x,x_i,\omega) \\ F_1^-(x,x_i,\omega) \end{array} \right] |_{z=0} = 0, \quad F_1^+(x_i,x_i,\omega) = 0, \quad F_1^-(x_i,x_i,\omega) = 0,
\]

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Figure 1 A complex, realistic, synthetic velocity model used in Elison et al. (2020) to study the impact of short period internal multiples. The red dashed line separates the overburden from the target area.

(b) Normal incidence reflection coefficient at $x = 0$ as a function of depth.

we can initialize the KB-scheme, such that each upward propagation yields an update to the focusing function $F$ and its vertical derivative $\partial_z F$. Here $x_\perp$ denotes the vector in a $z =$-const plane, and $\partial_1$ indicates that the vertical derivative is taken on the first spatial variable. The process is repeated until $z = 0$ (i.e., the surface) is reached. Here, the first term in eq. (2) denotes a delta function around time zero (i.e., a temporally and spatially band-limited spike) at the focusing position. In the following, we use a fourth-order Runge-Kutta method with a fixed step size.

Numerical Example

We consider a complex synthetic model previously studied by Elison et al. (2020) shown in Figure 1 and a focusing function corresponding to a focusing position on the red dashed line. Figure 2 shows the down-going components where (a) was modelled with the approach outlined above followed by the up-down decomposition. Fields in Figures 2(b) and 2(c) were produced by applying the augmented and conventional Marchenko method to the reflection data, respectively. The augmented Marchenko method outcome (see Elison et al., 2020, for more details) was wavefield extrapolated with a smooth model time-reversed direct transmission to produce the outcome in 2(b). Clearly the focusing functions in Figure 2(a) and 2(b) are nearly identical, apart from linear tapering artefacts present in Figure 2(b) and a very small phase difference which can be seen by looking at the normal incidence ray parameter trace (Figure 3(b)) as well as its amplitude and phase spectra (Figure 3(c-d), respectively). This reference result is thus very strong evidence that the augmented Marchenko method produces a nearly perfect focusing operator, since it correctly handles the fine layering in the overburden. The central trace comparison (Figure 3(a)) shows larger differences due to subtle differences at large ray parameters.

The modeled result quickly exposes the differences between the ideal result and one obtained from incorrectly constrained conventional Marchenko method. Said differences are most apparent in (but not limited to) the early part of the focusing function (in this example around -0.35 s), where the time window protects the direct arrival (see panels in Figure 2 and traces in Figure 3). These subtle differences and the impact they could have on the migrated image (see example in Elison et al., 2020) are a stark reminder of a) that a good quality benchmarking tool is important to have and b) the augmented Marchenko method, in particular the band limited minimum phase reconstruction step, could still benefit from further refinements.

To further evaluate the fidelity of our approach we truncate the model in Figure 1 below the overburden-target boundary (dashed red line) and model a wavefield due to a dipole source at the redatuming level $\partial D_1$ and monopole receivers at the acquisition surface $\partial D_0$ using an open source time domain finite difference wavefield modeling code by Thorbecke and Draganov (2011). The outcome is an upward transmission which we subsequently multidimensionally convolve with $F^+$, the down-going component of the focusing function. Physically this corresponds to extrapolating the measured transmission back to
Figure 2 Focal point gathers of the focusing functions for the model in Figure 1, with the redatuming level at $z = 1200$ m, computed using (a) the KB modeling scheme, (b) augmented and (c) conventional Marchenko method. (d) Finite difference modeled transmission response due to a dipole source at $z = 1200$ m and receivers at the surface.

Figure 3 (a) central trace of gathers 2(a-red,b-blue,c-black). (b) idem for the normal incidence ray parameter, (c) amplitude and (d) phase spectra of traces in (b).

Figure 4 (a,b,c) Focal point gathers of the convolution of focusing fields Figure 2(a,b,c) with the transmission field in Figure 2 (d). Panels (d,e,f) are the FK amplitude spectra of gathers (a,b,c) respectively. The results are dressed with a square of the Ricker wavelet.
the redatuming level while correctly accounting for (and removing) all the internal scattering along the way. An alternative way to interpret this, by invoking some of the source-receiver reciprocity relations of the transmission operator, is an injection of $F^+$ at the acquisition level into the medium and measuring the response at the redatuming level. In either case, the outcome should be given by a spatially and temporally band-limited delta function. The resultant wavefields are shown in Figures 4(a-c). Clearly, both the augmented Marchenko focusing function as well as the one obtained using the KB modeling approach focus to a point, while the one from the conventional Marchenko method does not. This should of course not be surprising, however we would like to point out that the modeling approaches of the focusing functions and the transmissions, were fundamentally different and yet when combined together yielded a high fidelity result.

The analysis presented above can be extended to an arbitrary medium (within the evanescent field limitations mentioned above) and should be of great help when further developing correct constraints to the Marchenko equation in complex geological settings (see Elison (2020) for a complex laterally varying overburden example and further details). In particular, we believe that this work should provide high quality reference solutions for other approaches where the direct focusing function cannot be easily determined (Vasconcelos and Sripanich, 2019) and for the 2D augmented Marchenko methods, where implementation of the multidimensional minimum phase reconstruction is an outstanding theoretical challenge (Dukalski, 2020; Peng et al., 2021).

Conclusion

We have shown that wavefield depth extrapolation is suitable to accurately model focusing functions. KB-modelled focusing functions show that the augmented Marchenko equation determines focusing functions which correctly handle short-period internal multiples (for a horizontal overburden). The availability of reference focusing functions allows us to evaluate the performance of the augmented Marchenko method beyond laterally invariant models, as well as methods relying on other constraints.

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References